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Trends in Efficiency of Angolan Commercial Banks Outside the Top Ten: A DEA Window Analysis

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Abstract: The objective of this study is to analyze the trend in the relative efficiency levels of Angolan commercial banks, whose asset size means that they are not included in the group of the ten largest. The DEA methodology was adopted, applying input-oriented radial models, namely: CRS and VRS, based on the efficiency averages obtained between windows. The sample included banks that, due to the small size of their assets, their performance has not been subject to systematic analysis. The results obtained indicate that, although the average of the pure technical efficiency scores is higher than the average of the technical efficiency of scale scores, in the period (2016-2023), pure efficiency shows an upward trend, contrary to scale efficiency which shows a downward trajectory, and that the uncertainty associated with the observance of the identified trends is greater for the average scores obtained for scale efficiency than for pure efficiency. Within the scope of commercial policies aimed at increasing banking efficiency, this study can contribute to a more informed view, which allows decision-making, the basis of which goes beyond the traditional standard profit function, where only the profitability of banks is considered.

Keywords: Angolan banks; Efficiency; DEA; Window analysis.

1. Introduction

In their role as financial intermediaries, banks play an important role in any economy by allocating funds between different economic agents. This role, which is also one of the main sources of revenue for banks, represents one of the most consistent ways in which banks acquire and consolidate their assets over time. Thus, their size is important, as it gives them the ability to be more resilient to adverse events inherent in the banking business.

The size of bank assets, such as the volume of resources held by them, from which they generate profits, i.e., seek to be efficient, is an indicator that has been the basis for assigning scores that classify them according to their size. However, the size of their assets does not always accurately reflect the dynamics of the banks' activity, present in the levels of productive efficiency that they experience.

The analysis and importance of banking efficiency, as a way of understanding how banks optimize productive resources, is not a recent concern, a fact attested by several pioneering studies (1).

The efficiency of banks depends on factors that directly or indirectly affect cost absorption and service provision. Over the past 20 years, these factors have been the subject of extensive analytical and empirical literature, driven not only by the major financial crises of recent decades but also by structural changes in the sector (2). Such discoveries have led financial sector firms in general, and banks in particular, to resize the scale of their operations.

In Angola, the cases of the banks BCS (Banco de Crédito do Sul) and Banco YETU, integrated in the SADC banking association, can be cited, which demonstrate that they are making a greater effort to internationalize their activities, increasingly focused on conquering new markets, aiming at possible gains both by optimizing their resources (operational gains) and by increasing the size of their operations (economies of scale), that is, seeking to be more efficient. The bibliometric analysis of the literature on the evaluation of the efficiency of commercial banks in Angola, in the period between 2008 and 2024, from authors such as: (3) (4) (5) (6) (4) (7) and (8), reveal the absence of studies on the evaluation of banks classified as small. This record, normally associated with the systemic importance attributed to banks with larger volumes of assets, and their role in the

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stability of the financial system as a whole (9), may be the basis for the lack of research on the efficiency of banks with relatively smaller assets.

In the literature related to the evaluation of commercial banks, it is common to find studies based on the analysis of economic and financial indexes that provide insights into efficiency levels based on the profitability of equity aggregates and results. However, from a non-metric approach, DEA emerges as a widely used alternative in several analyses on the trend of efficiency scores, such as: the production, asset, value-added approaches and the financial intermediation approach, adopted in this study.

This growing use of the DEA methodology in the process of assessing banking efficiency is mentioned in several articles on bibliometric analysis of DEA production and, in this sense, the work of (10). Traditionally, banks were assessed only considering their financial performance indicators. However, the adoption of this methodology, based on the use of aggregated indicators (ratios), does not always allow for the effective capture of the individualized effect of variables that are difficult to measure at the outset, as is the case with the measurement of the impact of management on the operational performance of banks. Therefore, the DEA approach offers several advantages in contrast.

The possibility that this offers to introduce several inputs and outputs simultaneously, obtaining individual efficiency scores for each of the DMUs under analysis, which may have different units of measurement; it is capable of interacting with both qualitative and quantitative issues; it allows the identification of inefficiencies, constituting an effective decision-making tool by directing management's attention to areas that can be improved; it dispenses with (but can tolerate) prespecifications of underlying production functions; it can consider systems of preferences of evaluators and managers, assigning weights to the various resources based on the calculation of efficiency, which may reflect certain perceptions of decision-makers regarding the relative importance of certain inputs in the volume of outputs; it is based on individual observations and not on average values, since it uses data already observed (11) and (12).

The objective of this study is to analyze the trend in the efficiency averages of the banks under analysis, based on the efficiency averages obtained between DEA windows, so that the results can allow a more informed understanding of the efficiency patterns of Angolan commercial banking. Given the more informed changes, both in the way management processes and, in the technology, used by banks, over time, it is expected that their performance tends to increase. Thus, the study is justified by the need to seek a more global understanding of the trends in commercial banking efficiency in Angola, providing the information base that can guarantee the taking of managerial and strategic decisions that make Angolan commercial banking more efficient in the long term.

2. Literature Review

2.1. Assessment of the relative efficiency of small banks

It is common in the banking sector for the size of banks to be used to measure the degree of economies or diseconomies of scale, in a given context of efficiency assessment (13).

Since smaller banks generally have a smaller volume of assets, it is expected that the scale of their operations will be more prone to changes, with an impact on the volatility of their efficiency levels. In this sense, the operations of small banks may be more subject to variable returns, so radial models that objectively assess the efficiency of DMUs, whether by minimizing costs or maximizing results, may allow for a better capture of variations in efficiency, based on the analysis of weights based on management, technology or even regulatory aspects, as mentioned in the introduction.

Economists' concern with evaluating the performance of small banks is not new, given their important role in the allocation of credit to small and micro-enterprises, representing an important source of job creation (14).

In Angola, the economic and financial context in which banks operate is particularly difficult, due to considerable levels of informalization of economic activities, financial illiteracy and asymmetric information among economic agents. The process of evaluating smaller banks is more difficult since the aforementioned framework makes them even more heterogeneous. This framework is also corroborated in the financial literature, according to which the performance of small banks, compared to large banks, is affected by differences related to the composition of assets and liabilities, expenses, revenues non-financial, capital ratios, competition and access to credit information, which provide a certain degree of complexity in measuring their efficiency (9).

Considering certain conditions such as their location (rural or urban) or the type of technology used, small banks may be more efficient, regardless of their profitability. This finding is supported by (15), which states that the standard profit function treats large and small banks as if they should have the same variable results, in the presence of the same input and output prices. However, the same authors reinforce the idea that, based on this approach, there is a strong probability of a scale bias occurring, since larger banks have higher profits that are not explained by exogenous variables

(such as context variables). However, several studies evaluate banks, considering their size as an indicator of efficiency, as is the case of (16), (17) and (18).

A given DMU (Decision Making Unit) is considered efficient if it is not possible to reduce any of the inputs without reducing other outputs or to produce more of any of the outputs without increasing other inputs. (19) and (20), therefore, the possibilities offered by models will be presented below, which allow the evaluation of the efficiency, based on observed data, of variables of interest in the banking context, outside the scope of ratios.

2.2. Assessing the efficiency of commercial banks in Angola vs. Africa: A DEA approach

In the Angolan context, there is not much literature on the evaluation of production units using a DEA approach, and it is even less extensive when the topic is related to the evaluation of the efficiency of commercial banks, as mentioned in the introduction. The literature review allowed us to subdivide the studies into five groups, taking into account the type of DEA models used, namely: radial (VRS and CRS), Malmquist-DEA, B-Convexity, Fuzzy-DEA and Stochastic-DEA. To present the context of the use of DEA models in the evaluation of commercial banks in Angola, the main ideas contained in the reviewed articles are presented below. The bibliometric analysis did not allow us to find any DEA study that had analyzed the efficiency trend in the Angolan banking sector using a DEA window approach. In this sense, studies such as those by (5) (7) (8) evaluate the relative efficiency of banks in Angola using DEA CRS and VRS models. In these studies, based on a financial intermediation approach, the authors generally develop the research in two phases, first obtaining results from the CRS model and in second from the VRS model. In the Angolan context, studies using modified DEA models in the evaluation of banks appear in studies by Barros, Liang and Peypoch, (4) using an innovative production frontier model, the B-Convexity model, where the results indicate that efficiency increases over the observation period, taking into account international experience, market share and local markets; (21) who used the Fuzzy-DEA and Stochastic-DEA models to assess how the randomness and imprecision underlying the data interfere with efficiency levels; (6) used a Bayesian stochastic frontier model to analyze the differences in efficiency between foreign banks, public banks, large banks and banks belonging to a local conglomerate; (3) who used the Malmquist-DEA model to identify the productivity growth of Angolan banks.

In a complementary manner, other studies on the analysis of the efficiency of banks in Africa were reviewed, which, due to their scope, could in some way reflect the state of the art about the evaluation of banks from a non-parametric approach (DEA). Such were the cases of (22), where the efficiency of African banks, included in the top 100 of the African Business magazine of 2006. The study makes an analysis based on the CRS and VRS models, which allowed the identification of reference pairs between the banks and a sensitivity analysis that allowed the identification of slack in the optimization of the variables; (23) which estimated the relative efficiency of 58 commercial banks, belonging to 5 East African countries, applying radial models (CRS and VRS), and arriving at results that indicate that the source of inefficiency revealed, originates in the process of resource optimization; (24) who assessed the financial and social efficiency of 18 small banks from 8 member countries of the Southern African Development Community (SADC) , where, from a production approach, they used an input-oriented DEA VRS model, which allowed obtaining results that point to higher financial efficiency indexes than social efficiency indexes; (25), it was reviewed as a reference to Mozambique, with characteristics similar to Angola, with regard to the configuration of the economic-financial system (countries speaking the same language, independent in the same year, opted for centralized production systems after independence, which also changed in the same phase to market economies). The study assesses the efficiency of 16 commercial banks operating in Mozambique, based on a non-radial VRS model based on slack, oriented to output (SBM-O). The results obtained revealed an average lower than 1, which generally classifies Mozambican commercial banks as inefficient.

2.3. CRS and VRS models: An overview

The review of the evaluation of commercial banks in Angola and the reviewed African countries makes evident the predominance of the use of radial DEA models (CRS and VRS).

The CRS and VRS models frequently appear in research on banking efficiency, given their ability to capture changes in banks' operational processes.

Considering the dynamic environment in which banks operate, the VRS model is often used since scale can have an impact on productivity, while the CRS may be suitable for global efficiency comparisons. In the study, the choice of models was based on the objectives of the analysis, i.e., the two radial models presented were adopted.

The possibilities offered by DEA as a methodology that allows for the assessment of efficiency have been the basis for several studies related to the evaluation of commercial banks. This

tool is particularly important in the Angolan context, where aggregated data on the performance of small banks are difficult to access and are not always validated by an independent entity, which is particularly relevant in the case of small banks.

The evaluation of efficiency, based on the radial DEA methodology, allows the isolation of particular effects, related to how the management of banks, as well as the context in which they operate, influence their performance, which is why it becomes even more relevant when it comes to the evaluation of small banks in Angola. Thus, based on evaluations of observed data, DEA provides a post-factum view, useful for future findings and corrections that may contribute to improving the optimization levels of commercial banks in general and of smaller banks in particular. From this point of view, DEA as a methodology, composed of techniques and models, aimed at evaluating the efficiency of DMUs, finds in the radial models (input or output-oriented), its initial versions which are the subject of review in this study, namely: the CRS and VRS models, initially proposed by (26).

Considering the justification that appears in the topic related to the research methodology, only the mathematical formulations for the input guidelines will be presented, starting from (27).

In the context of DEA, radial models are represented by a group whose orientation is constituted by the input/output matrix, and in these models, the condition is assumed that the inputs must be minimized and the outputs maximized (28). The aforementioned condition defines that the variations that occur in the quantity of one of the components of the input/output matrix are directly proportional to these, that is if the quantity of input varies by one unit, the same variation will occur in the output and vice versa, if constant returns to scale are considered.

According to (29) the CRS model, it represents a generalization of Farrel's definition (1957), based on the construction of a non-parametric linear surface, involving the data.

The consideration of the assumption of constant returns implies the assumption that all DMUs operate at the same production scale, therefore, according to (30), the assumptions of the CRS model, based on the existence of a linear relationship between inputs and outputs, are only justified when all DMUs are operating at an optimal scale.

The mathematical formulations for the CRS and VRS models were taken from (27).

Considering that the restrictions applied to DEA models presuppose the consideration of maximum and minimum values, for the formulation of the CRS model with input guidance, we have that:

$$\text{Maximizer } h_k = \sum_{r=1}^{s} u_r y_{rk} \quad (1)$$

Subject to:

$$\sum_{r=1}^{m} u_r y_{rj} \quad (2)$$

$$\sum_{i=1}^{n} v_i x_{ik} = 1 \quad (3)$$

$$u_r , v_i \geq 0 \quad (4)$$

Where: y: inputs; x: outputs; r:1,…, m; i:1,…, n; j:1,…, N.

Similar to other simplified representations of reality, the DEA technique, to solve the problem of calculating efficiency in companies, underwent several changes with the inclusion in the initial model of the condition of non-constant returns (8).

In this sense, the VRS model differs from the CRS model in the type of predicted returns, this means that the variations that occur in one of the components of the input/output matrix do not result in a proportional variation in any of the components (input or output) of the matrix, that is, in this model the returns are variable.

The model measures the resulting combination of observed values for inputs and outputs, under the condition that, not being constant, the returns to scale assume the conditions of: increasing, constant or decreasing (31).

Among some of the reasons that lead to the adoption of models that predict variable returns to scale is the observation of negative values in both inputs and outputs, in addition to there being differences between DMUs, in their operational context, resulting from changes in the technology used, in regulatory aspects or in the financing conditions of their activities, causing the premise of observing constant returns to be questioned.

Given the maximum and minimum constraints, the formulation of the input-oriented VRS model is as follows:

$$\text{Maximizer } h_k = \sum_{r=1}^{m} u_r y_{rk} - u_k \quad (5)$$

Subject to:

$$\sum_{i=1}^{n} v_i x_{ik} = 1 \quad (6)$$

$$\sum_{i=1}^{m} u_r y_{rj} - \sum_{i=1}^{n} v_i x_{ij} - u_k \leq 0 \quad (7)$$

$$u_r , v_i \geq 0 \quad (8)$$

Wherw: y: inputs; x: outputs; u e v: pesos; r:1,…, m; i:1,…, n; j:1,…, N.

However, like any methodology, DEA also has limitations in its use for evaluating efficiency, some of which can be highlighted:

Because it is a linear programming technique, DEA observes each input and output only once, which can cause errors in its data, leading to biased results, since they can represent either efficient or falsely efficient solutions (12);

Lack of discriminatory power between DMUs. This disadvantage of DEA is also mentioned by (32), who states that degeneration is a very common problem in DEA models, due to the very structure of their linear programming problems, where there is a large number of redundant constraints related to inefficient DMUs and which, in some cases, causes there to be multiple optimal solutions;

According to (33) there is a consensus among operations research scholars that, because they do not estimate efficiency scores around an average, DEA models are sensitive to outliers and computation errors.

2.4. DEA Window

The dynamics in the trend of companies' efficiency records can reveal their causes and indicate directions for improving their performance, therefore, to capture the trend of efficiency scores of the two models presented (CRS and VRS), the DEA window analysis technique was used.

In addition to not requiring any assumptions to be made about the distribution of the data, another of the main advantages of DEA is that it works relatively well with small samples. (34), a situation verified in this study.

The DEA window technique consists of grouping the DMUs belonging to a given sample into overlapping analysis windows. This procedure allows the sample to be subdivided according to the principle of moving averages, which allows new data to emerge in each window.

		Periods							$E_{i\bullet}$
DMU_k		Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Row average
Windows	Window 1	$E_{1\,1}$	$E_{1\,2}$	$E_{1\,3}$					$E_{1\bullet}$
	Window 2		$E_{2\,2}$	$E_{2\,3}$	$E_{2\,4}$				$E_{2\bullet}$
	Window 3			$E_{3\,3}$	$E_{3\,4}$	$E_{3\,5}$			$E_{3\bullet}$
	Window 4				$E_{4\,4}$	$E_{4\,5}$	$E_{4\,6}$		$E_{4\bullet}$
	Window 5					$E_{5\,5}$	$E_{5\,6}$	$E_{5\,7}$	$E_{5\bullet}$
$E_{\bullet j}$	Column Average	$E_{\bullet 1}$	$E_{\bullet 2}$	$E_{\bullet 3}$	$E_{\bullet 4}$	$E_{\bullet 5}$	$E_{\bullet 6}$	$E_{\bullet 7}$	$E_{\bullet\bullet}$

Figure 1. Default DEA window configuration (35)

The DEA window technique is a dynamic analysis that observes selected DMUs over a period of time, which allows for the increase in analyzed DMUs (36).

Based on the ideas contained in (37), about the configuration of the (X_{kw}) input and output matrices (Y_{kw}) of each window (kw), these can be presented in the following forms:

$$X_{kw} = \begin{bmatrix} x_1^k & x_2^{k2} & \cdots & x_N^k \\ x_1^{k+1} & x_2^{k+1} & \cdots & x_N^{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{k+w} & x_2^{k+w} & \cdots & x_N^{k+w} \end{bmatrix} \quad (9)$$

and

$$Y_{kw}=\begin{bmatrix} y_1^k & y_2^{k2} & \cdots & y_N^k \\ y_1^{k+1} & y_2^{k+1} & \cdots & y_N^{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_1^{k+w} & y_2^{k+w} & \cdots & y_N^{k+w} \end{bmatrix} \quad (10)$$

According to (38) in the analysis that applies the DEA window technique, carried out type analysis (k - p + 1) separated, in which each analysis examines new DMUs where; n: number of DMUs; k: number of periods to which the data refer; p = window width ($p \leq k$) and w: number of windows, i.e. there are n DMUs × k (number of periods). Thus, Table 1 presents the mathematical formulations related to the operationalization of the DEA window analysis.

Based on the information contained in Table 1, in the topic referring to the methodological paths followed, the procedures adopted to define the width of the analysis windows are presented, as well as their justification, that is, the largest possible number of different DMUs.

3. Materials and Methods

To analyze the dynamics of efficiency, in this article, the two radial models presented (CRS and VRS) were used, both with input orientation. The orientation given to the models arises from the consideration that inputs are the variables over which companies have the greatest capacity for rationalization and, consequently, optimization.

The CRS model was used with the intention of capturing the trend of efficiency scores in the process of combining productive resources by the banks analyzed, while the VRS model was used to capture the effect of the technology used on their efficiency throughout the period analyzed.

Considering the theoretical assumptions for determining the width of the analysis windows, contained in (38) and presented in Table 1, a window width of four years was adopted. This procedure allowed a considerable increase in the number of different DMUs, as opposed to the 9 (8) = 72 DMUs that would be available if the evaluation had been carried out separately for each of the DMUs in each of the eight periods. Thus, the configuration of the analysis windows, presented in Table 2, allowed an increase in the discriminatory power for 180 different DMUs analyzed, as demonstrated in Table 1.

To select the variables targeted for analysis, the typology of variables provided for in the financial intermediation approach was considered, seeking a balance between the number of inputs and outputs about the number of DMUs, to obtain a good discriminatory power of the efficiency scores, throughout the analysis windows, allowing the number of efficient DMUs to be as small as possible in each analysis window. Like other less advanced countries, whose economies are in a phase of transition towards industrialization, the Angolan financial system is mainly made up of commercial banks, whose basic function is to capture deposits and grant loans, a view corroborated by (39) the fact that this approach is particularly appropriate for banks whose basis of operations consists of transforming deposits and funds acquired from other financial institutions into loans and financial investment.

In this sense, the rule suggested by was applied (40), according to which the number of DMUs analyzed must be at least twice the total number of inputs and outputs selected.

Still aiming to give greater discriminatory power to the efficiency scores obtained, a correlation test was carried out in order to guarantee a selection of variables with a lower degree of association between them, and therefore, with greater explanatory capacity between them.

Table 1. Criteria for defining the width of analysis windows

Window specifications	Formulas	p = 4 years	p = 5 years
Number of windows	w = k - p +1	8 - 4 + 1 = 5	8 - 5 + 1 = 4
Number of DMUs in each window	(n) (p)	9 (4) = 36	9 (5) = 45
Number of different DMUs	(n) (p) (w)	(9) (4) (5) = 180	(9) (5) (4) = 180
Variation in the number of DMUs	n (p - 1) (k - 1)	(9) (3) (7) = 189	(9) (4) (7) = 252

Considering the premises presented for the configuration of the DEA windows, the configuration presented in Table 2 was considered.

Although a window with a width of five years also allows obtaining the same number of different DMUs, it is in the case where p = 4 years that the number of DMUs registers a lower value in its variation, indicating greater stability in terms of the results to be obtained in this condition, that is, opting for a window with a width of 4 years obtains a variation in the number of different

DMUs, 72% lower compared to the variation that occurs when the adopted window is 5 years, as shown in table 1.

Table 2. Analysis window configuration

Periods	1	2	3	4	5	6	7	8
Window 1	2016	2017	2018	2019				
Window 2		2017	2018	2019	2020			
Window 3			2018	2019	2020	2021		
Window 4				2019	2020	2021	2022	
Window 5					2020	2021	2022	2023

4. Data and Variables

The decision to analyze only small banks finds its theoretical justification in studies by (15), allowing the analysis of the efficiency trend of banks with a relatively low volume of assets, regardless of their profitability, in the period studied. As in most evaluations within the scope of nonparametric DEA modelling, the generalization of the results obtained will be carried out within the strict scope of the sample, since a post-factum evaluation is carried out, that is, carried out based on data already observed, for each of the variables.

Data relating to the economic and financial performance of nine Angolan commercial banks were analyzed, with all published data referring to the period analyzed (2020-2023). Banks whose volume of assets does not exceed AOA 250,000,000 were analyzed, as illustrated in graph 1.

The data were obtained from the reports and accounts of the respective banks, from publications on their official websites, appearing referenced in: (41) (42) (43) (44) (45) (46) (47) (48) and (49), as shown in Table 3.

Table 3. The set of banks analyzed

Acronym	Designation	Established	Swift code
BCA	Banco Comercial Angolano, S.A.	1999	COMLAOLU
BE	Banco Económico, SA	2002	BESCAOLU
BNI	Banco Internacional de Negócios, S.A.	2007	BNICAOLU
BCH	Banco Comercial do Huambo, S.A.	2010	BCHUAOLU
BVB	Banco Valor, S.A.	2010	BVBXAOLU
YETU	Banco YETU, S.A.	2015	YETUAOLU
BCS	Banco de Crédito do Sul, S.A.	2015	CDTSAOLU
BIR	Banco de Investimento Rural, S.A.	2015	BIRVAOLU
BOCLB	Banco da China Limitada - Luanda	2017	BKCHAOLU

The data that make up the sample analyzed are covered by the period 2016-2023 and belong to Angolan commercial banks, which in the ABANC (Angolan Banking Association) classification (50)**,** do not appear in the top ten positions, more specifically with an asset value not exceeding. Therefore, banks whose equity value is less than AOA 250,000 were included in the sample, as shown in graph 1.

According to (15), except after long periods of growth, mergers and acquisitions, small banks can become large, so, since the period analyzed is relatively short, it would not be expected that the impact of these on the efficiency of banks would be significant.

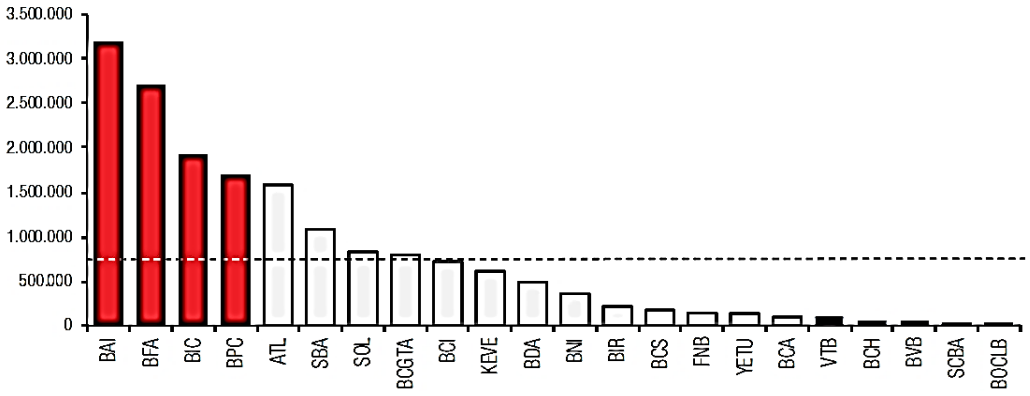
In this sense, during the time interval analyzed (2016-2023), the banking system registered some mergers and acquisitions between smaller banks, this being the main basis for the temporal delimitation of the research. This occurred between the banks Standard Chartered, S.A. and Finibanco Angola, acquired by Access Bank Plc., from Nigeria, in 2022, Banco Atlantico (established in 2016, as a result of the merger between Banco Privado do Atlantico and Millenium BCP Angola) and Banco da China Limitada - Luanda (established in 2017, as a result of the establishment of commercial relations at an institutional level between the Republic of Angola and the People's Republic of China), according to the information contained in (51) and (52), respectively.

Finally, VTB Africa bank, due to the absence of published financial statements in 2023, was also not analyzed. However, since, within the scope of DEA, it is an assessment of the relative

efficiency of DMUs, the results were not affected, since their scope is exclusive to the banks included in the sample.

The restriction of the sample to a smaller universe, with the exclusion of larger banks, produced repercussions on its size. However, the overlapping windows technique allowed the increase of different DMUs, as mentioned in the topic related to the DEA window.

The criteria followed by (50)**,** to classify the set of commercial banks existing in the Angolan financial system, the size of their assets is considered as the main indicator, as can be seen in graph 1.



Graph 1. Size of credit financial institutions by assets in AOA´000 (50)

At the time of preparation of this study, the available data referred to the year 2023. The time horizon of the report is also relevant, as it coincides with a period of restructuring and mergers that gave rise to new banks, namely: BE, BVB and BIR. In line with the literature consulted, namely, (53) and (34), the following variables were adopted as output (loans to customers and operating revenue excluding interest) and as input (customer deposits and personnel costs).

Thus, in Table 4, the descriptive statistics of the mentioned variables are presented, highlighting the presence of extreme minimum values, however compatible with the standard deviation indicators, which in the set of variables, did not exceed half of the values of the corresponding averages.

Table 4. Descriptive statistics of variables (in AOA ´000)

Variables	Minimum	Maximum	Average	St. Deviation
Deposits {I}	1	1,589,389,247	191,075,688	349,312,843
Personnel costs {I}	415	16,807,226	4,274,780	4,256,062
Loans {O}	1	178,818,324	33,126,499	45,113,393
Operating profit without interest {O}	232	178,470,270	16,373,725,640	34,547,148
Deposits {I}	1	1,589,389,247	191,075,688	349,312,843

The results, which appear in Table 5, provide information that there is a positive correlation between the input and output variables. They also provide evidence of the independence between the input variables, which is the same for the output variables, ensuring the appropriate level of robustness in the efficiency scores obtained.

Table 5. Correlation matrix between input and output variables

Variables	Deposits	Personnel costs	Loans	Operating profit without interest
Deposits	1,000			
Personnel costs	0.865	1,000		
Loans	0.684	0.762	1,000	
Operating profit without interest	0.867	0.775	0.531	1,000

5. Results and Discussion

The results were obtained using the Efficiency Measurement System (EMS) program (54). Table 6 presents an overview of the scores obtained for each of the banks in the sample, in each year and for each of the proposed models.

The scores represent the efficiency averages for each of the years and for the years as a whole (2016-2023).

In alignment with (55), the results obtained were as expected, since the scores under the VRS assumption are greater than or equal to those obtained under the CRS assumption.[1]

Table 6. Overview of efficiency scores obtained

Model	Year	BCA	BE	BVB	YETU	BCS	BOCL B	BIR	BCH	BNI	Mean
CRS	2016	13%	18%	9%	6%	7%	100%	3%	57%	23%	26%
	2017	26%	25%	19%	15%	41%	51%	9%	80%	26%	32%
	2018	69%	74%	19%	47%	72%	8%	100%	30%	50%	52%
	2019	72%	83%	17%	50%	67%	1%	34%	87%	41%	50%
	2020	3%	59%	35%	64%	100%	100%	66%	77%	55%	62%
	2021	14%	86%	26%	97%	45%	84%	47%	50%	62%	57%
	2022	7%	14%	37%	92%	45%	76%	30%	29%	46%	42%
	2023	11%	100%	18%	49%	30%	26%	32%	34%	51%	39%
	Mean	27%	57%	23%	53%	51%	56%	40%	56%	44%	45%
VRS	2016	85%	100%	39%	22%	71%	100%	100%	99%	100%	80%
	2017	67%	100%	36%	29%	62%	100%	99%	100%	94%	76%
	2018	100%	100%	29%	69%	100%	48%	100%	48%	100%	77%
	2019	100%	100%	27%	64%	97%	43%	46%	100%	76%	73%
	2020	13%	64%	38%	66%	100%	100%	79%	88%	92%	71%
	2021	20%	89%	36%	98%	50%	86%	58%	74%	100%	68%
	2022	21%	21%	41%	100%	60%	87%	33%	57%	99%	58%
	2023	19%	100%	38%	53%	59%	61%	39%	62%	100%	59%
	Mean	53%	84%	36%	63%	75%	78%	69%	79%	95%	70%

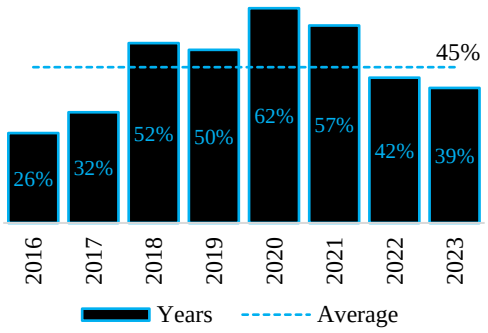
The period analyzed covers the beginning and end of an economic recession, which lasted around five years and which, from 2021 onwards, began to slow down, with signs of economic growth beginning in 2022 (56). This scenario had a structural impact on the performance of the banking sector in Angola, mainly due to the impact of the depreciation of the exchange rate on the financial results of banks, which may be the reason why none of the banks analyzed recorded average efficiency values between 2016 and 2023. This finding is supported by (57), according to which, in 2019, in the Angolan banking sector as a whole, as a result of the 73% drop in exchange rate results, net results also decreased, as did overdue loans, which increased by 34 % in the same period. Some of the strategies adopted were precisely the restructuring of the banks' shareholder structures, either through mergers or acquisitions, as mentioned.

5.1. Pure technical efficiency

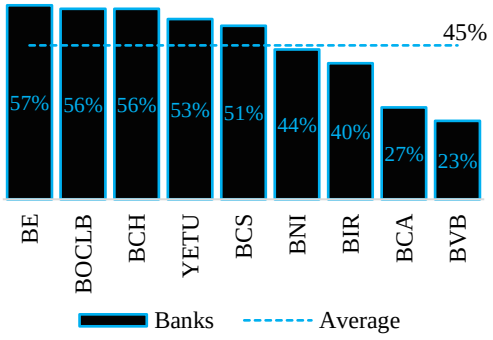
Graphs 2 and 3 show the pure technical efficiency scores, by bank and year. Thus, from graph 2, it can be seen that the variations around the average, for all banks in the sample, registered their lowest point in 2016 and their highest point in 2020. These variations revolve around an average of 45%, still below the 50% that would be half the efficient average. A visual inspection of graph 3 confirms this, allowing us to see that, during the period analyzed, no bank in the sample was efficient, with BE being the least inefficient.

The results obtained are in line with those presented in (56) and (57), coinciding with the periods of lowest net results in Angolan banking.

[1] An overview of these results can be found in appendices A and B at the end of the text.



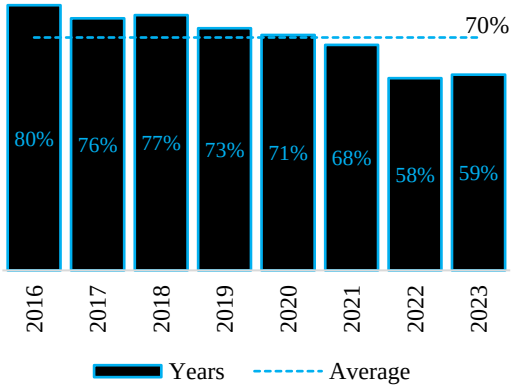
Graph 2. CRS vs. Average efficiency (years)



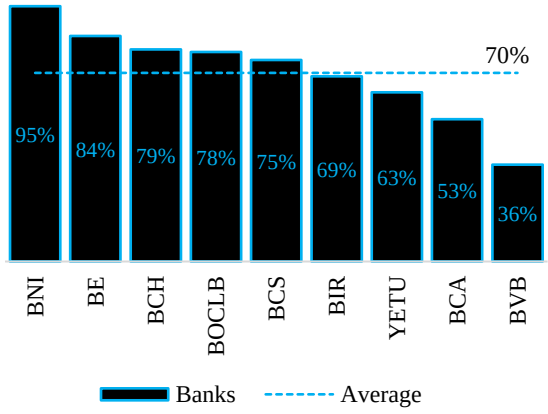
Graph 3. CRS vs. Average efficiency (banks)

5.2. Technical efficiency of scale

From the observation of the results of technical and scale efficiency, presented in graphs 4 and 5, it is possible to verify that, despite being equally inefficient over the years and for all banks, the average scale efficiency score is 25% higher than the average recorded in pure technical efficiency. This result is due, in part, to the fact that more than half of the banks analyzed recorded efficiency scores above 74%, that is, very close to the efficiency frontier, indicating that about half of the banks analyzed operate close to their optimal size.



Graph 4. VRS vs. Average efficiency (years)

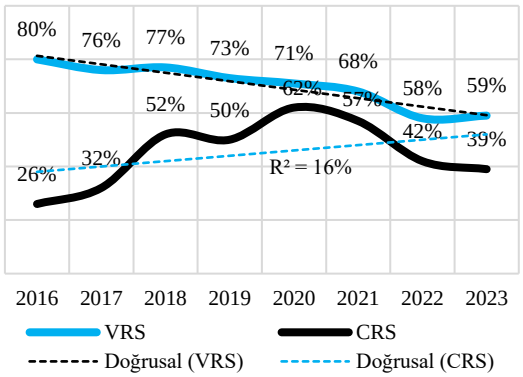


Graph 5. VRS vs. Average efficiency (banks)

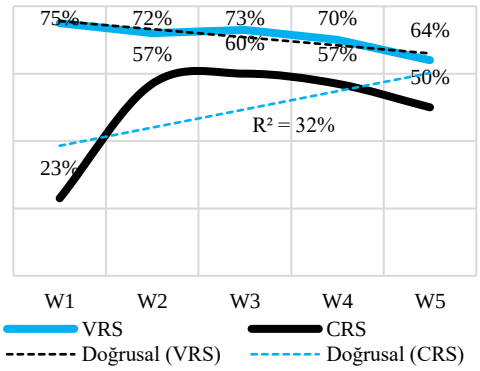
5.3. Comparative analysis between pure technical efficiency and technical efficiency of scale

Regarding the combined analysis of the trend of pure technical efficiency and scale efficiency, two important points are worth highlighting: first, the CRS model allows us to evaluate the performance of DMUs in the process of optimizing productive resources; and second, the VRS model allows us to evaluate the performance of DMUs in the process of optimizing available technological capacity. Thus, from graphs 6 and 7, it is possible to observe that, throughout the period analyzed, the VRS curve presents average efficiency scores that are higher than those recorded in the scores that make up the CRS curve. This behavior of the two curves indicates that, on average, the set of banks analyzed recorded better performance in indicators related to the optimization of the technological capacities installed in the sector, as well as the size of their operations. In contrast, the lower average performance recorded in the CRS scores indicates that the greatest sources of inefficiencies recorded in the period and for the banks analyzed are related to probable operational failures related to their management processes.

However, the trend lines of the two efficiency dimensions analyzed point to long-term convergence, beyond 2023. This convergence trend, which begins in the middle of the second window, is explained by the beginning of a downward trajectory in the average efficiency scores, both for pure efficiency (CRS) and for scale efficiency (VRS), from the beginning of the third window. Finally, it is worth noting that, despite always recording higher scores than those recorded for pure efficiency scores, the trend in the scale efficiency score curve was always in the direction of its decrease.



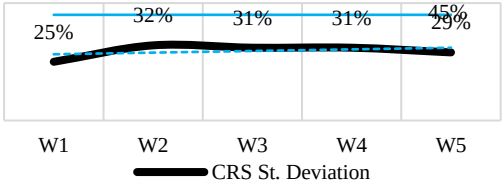
Graph 6. Efficiency averages per year



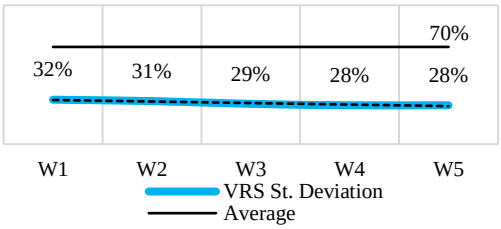
Graph 7. Efficiency averages per window

As expected, the trends in efficiency scores appear more linear across the five analysis windows. However, in graphs 8 and 9, the idea of an upward trend in pure technical efficiency scores and a downward trend in technical efficiency scores is reinforced, albeit more gently, in the perspective between the windows presented.

Considering the minimum and maximum values for the deviations in the efficiency scores, both pure and scale, whose results appear in graphs 8 and 9, it is possible to observe that the level of variation in the scale efficiency scores is greater than that recorded in the variations in the pure efficiency scores. This is evident by the record of higher standard deviations and, therefore, further away from the average of the efficiency scores (70%). These records may also indicate the existence of a greater degree of uncertainty regarding the trend of evolution of the pure efficiency scores, for the set of banks analyzed, in the coming years.



Graph 8. Deviation from mean efficiency (CRS)



Graph 9. Deviation from mean efficiency (VRS)

Unlike efficiency scores obtained annually, those obtained on a window basis allow for smoothing the records in the distances between the efficiency scores and their averages. This procedure allows for a more precise capture of the dynamic effects present, both in the trend of pure technical efficiency and in the efficiency of scale; an effect associated with the uncertainty present in the future behavior of the respective trends, which can serve as a predictive element for decision-making by the banks analyzed, both at the operational level and in defining the size of their business.

6. Conclusions and Observations

Although between the years (2016 and 2023) the average efficiency scores for the set of banks analyzed were 70% for scale efficiency and 45% for pure efficiency, the results obtained allowed us to identify that, while the evolution of pure technical efficiency scores tends to be decreasing, the opposite occurs with the trend in the evolution of technical scale efficiency scores. Thus, with a view to improving scale efficiency levels, one of the most suitable strategies is already underway, namely acquisitions and mergers, which allow for an increase in the volume of operations and a reduction in unit costs. However, corrective actions can be implemented, based on strategies that include: retaining and segmenting more customers to maximize revenue capture, improving financial management through capital restructuring, as well as taking advantage of new technologies to improve the customer experience in their relationship with new products and services provided by banks.

The analysis of the trend in the efficiency scores of nine of the smallest Angolan banks, in terms of the size of their assets, allowed us to identify a convergence between the levels of efficiency they achieved and the levels of risk associated with them.

Potentially, risks such as credit risks could be identified, precisely associated with their small size in terms of the assets they hold, to guarantee both the deposits they collect and the loans they grant.

Over the period analyzed, all banks recorded average scale efficiency levels higher than pure efficiency levels. However, the downward trend in scale efficiency indicates the need to develop actions to improve the optimization of the sector's technological capabilities.

The fact that pure efficiency recorded an average of 45%, 25% below the average for scale efficiency, also indicates that, in general, inefficiencies originate from banks' operational optimization processes, despite the record of an upward trend in their scores. Some of the strategies for improving pure technical efficiency would involve continuous training of employees to ensure greater results with the same resources, adoption of innovative practices that allow the automation of processes and consequent minimization of operational costs, in line with the input orientation of the models adopted in the study, or benchmarking. to adapt operations, aligned with the new and best trends in the sector.

In the period between (2016-2023) none of the banks were technically efficient, in any of the efficiency dimensions evaluated.

Being a preliminary study, the research does not cover all banks, as they do not belong to the group of the ten largest Angolan commercial banks, either due to insufficient data or due to changes in their ownership that led to the change of their brand.

As a line of action, it is proposed to use other measures that allow a better understanding of the performance of banks over time, such as: the Malmquist productivity index for benchmarking or the Luenberger index to capture operational constraints and external effects on bank management. The two measures applied to a larger sample could allow the identification of the causes that influence the performance trends of these banks, between different periods, namely the parallel and downward trajectory of the two efficiency dimensions from the third window onwards.

In terms of policies aimed at increasing banking efficiency, the assessment of only small banks in Angola, in the referenced period, can contribute to a more informed view of banking decision-makers, which allows decision-making, whose basis goes beyond the traditional standard profit

function, where only the profitability of banks is considered. In this sense, because the objective is the strict analysis of the average efficiency, based on numerical and statistical data, other aspects such as operational and strategic, which can allow a global analysis of the banks analyzed, such as: customer service, innovation, financial stability) were not taken into account, which presents itself as a limitation to the study and at the same time a direction for future research, based on sensitivity analysis, in order to provide a more detailed discussion on the validity and accuracy of the models used.

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Data Availability Statement: For access to data supporting results please contact to the authors of the article.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. CRS input orientation

Banks	2016	2017	2018	2019	2020	2021	2022	2023	Mean	St. Dev.
	13.46%	12.17%	27.75%	24.01%					19%	8%
		38.90%	89.76%	87.75%	3.24%				55%	42%
BCA			89.76%	87.75%	3.24%	13.09%			48%	47%
				88.14%	3.74%	13.12%	6.50%		28%	40%
					3.74%	14.70%	7.45%	10.73%	9%	5%
Mean	13.00%	26.00%	69.00%	72.00%	3.00%	14.00%	7.00%	11.00%	27%	28%
	17.72%	17.69%	48.31%	53.46%					34%	19%
		31.95%	81.17%	89.78%	56.48%				65%	26%
BE			93.11%	89.78%	63.63%	96.29%			86%	15%
				100.00%	65.95%	96.29%	13.97%		69%	40%
					51.30%	66.49%	13.44%	100.00%	58%	36%
Mean	18.00%	25.00%	74.00%	83.00%	59.00%	86.00%	14.00%	100.00%	57%	34%
	9.01%	10.04%	8.26%	7.01%					9%	1%
		28.65%	23.88%	19.31%	31.86%				26%	5%
BVB			25.30%	20.52%	34.14%	24.05%			26%	6%
				22.92%	36.04%	24.27%	33.72%		29%	7%
					38.14%	28.59%	39.65%	18.21%	31%	10%
Mean	9.00%	19.00%	19.00%	17.00%	35.00%	26.00%	37.00%	18.00%	23%	9%
	5.99%	5.61%	13.23%	13.60%					10%	4%
		25.08%	63.40%	62.20%	63.64%				54%	19%
YETUI			63.40%	62.20%	63.64%	95.60%			71%	16%
				63.30%	64.57%	96.53%	88.77%		78%	17%
					65.82%	98.66%	95.39%	49.40%	77%	24%
Mean	6.00%	15.00%	47.00%	50.00%	64.00%	97.00%	92.00%	49.00%	53%	32%
	7.00%	20.49%	15.38%	17.83%					15%	29%
		60.59%	100.00%	83.08%	99.07%				86%	29%
BCS			100.00%	83.08%	99.07%	43.16%			81%	29%
				84.09%	100.00%	43.31%	44.74%		68%	29%
					100.00%	48.54%	44.81%	29.51%	56%	29%
Mean	7.00%	41.00%	72.00%	67.00%	100.00%	45.00%	45.00%	30.00%	51%	29%
	100.00%	2.17%	0.45%	0.36%					26%	29%
		100.00%	9.99%	0.86%	100.00%				53%	28%
BOCLB			14.40%	0.86%	100.00%	80.17%			49%	27%
				0.87%	100.00%	85.40%	75.63%		65%	26%
					100.00%	85.40%	76.36%	26.25%	72%	25%
Mean	100.00%	51.00%	8.00%	1.00%	10.000%	84.00%	76.00%	26.00%	56%	25%
	3.08%	6.40%	100.00%	15.55%					48%	46%
		10.75%	100.00%	37.56%	60.65%				49%	38%
BIR			100.00%	40.31%	64.82%	44.23%			49%	27%
				43.66%	73.85%	51.81%	31.89%		48%	18%
					65.31%	45.41%	28.45%	31.50%	48%	17%
Mean	3.00%	9.00%	100.00%	34.00%	66.00%	47.00%	30.00%	32.00%	48%	31%

	56.92%	59.59%	16.28%	49.26%					49%	20%
		100.00%	35.01%	100.00%	73.37%				50%	31%
BCH			37.50%	100.00%	73.37%	47.57%			48%	28%
				100.00%	73.48%	47.65%	26.82%		47%	32%
					89.07%	56.24%	31.43%	33.84%	46%	27%
Mean	57.00%	80.00%	30.00%	87.00%	77.00%	50.00%	29.00%	34.00%	45%	24%
	22.66%	19.31%	24.58%	17.39%					21%	14%
		33.35%	60.00%	44.62%	50.04%				47%	13%
BNI			64.00%	47.47%	52.95%	59.43%			56%	13%
				55.86%	63.22%	68.44%	46.57%		59%	13%
					53.83%	58.27%	46.10%	50.98%	52%	12%
Mean	23.00%	26.00%	50.00%	41.00%	55.00%	62.00%	46.00%	51%	44%	12%
Mean	26%	32%	52%	50%	62%	57%	42%	39%		
St Dev	31%	23%	32%	32%	35%	32%	29%	28%		

Appendix B. VRS input orientation

Banks	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**Mean**	**St. Dev**
	85.35%	94.09%	100.00%	100.00%					95%	7%
		38.92%	100.00%	100.00%	5.76%				61%	47%
BCA			100.00%	100.00%	8.24%	16.44%			56%	51%
				98.54%	17.80%	20.93%	19.57%		39%	40%
					20.21%	22.78%	21.49%	19.29%	21%	2%
Mean	85.00%	67.00%	100.00%	100.00%	13.00%	20.00%	21.00%	19.00%	53%	39%
	100.00%	100.00%	100.00%	100.00%					100%	0%
		100.00%	100.00%	100.00%	65.50%				91%	17%
BE			100.00%	100.00%	65.50%	100.00%			91%	17%
				100.00%	67.65%	100.00%	21.06%		72%	37%
					59.34%	66.72%	20.46%	100.00%	62%	33%
Mean	100.00%	100.00%	100.00%	100.00%	64.00%	89.00%	21.00%	100.00%	83%	28%
	39.38%	42.32%	34.35%	25.85%					35%	7%
		30.26%	25.09%	20.05%	33.65%				27%	6%
BVB			27.44%	21.67%	35.33%	27.88%			28%	6%
				41.89%	38.10%	38.27%	37.03%		39%	2%
					45.88%	41.75%	44.03%	37.82%	42%	3%
Mean	39.00%	36.00%	29.00%	27.00%	38.00%	36.00%	41.00%	38.00%	36%	5%
	21.79%	24.36%	65.79%	62.22%					44%	24%
		33.68%	67.48%	62.99%	63.94%				57%	16%
YETUI			72.43%	64.15%	64.49%	96.36%			74%	15%
				65.89%	66.83%	96.68%	100.00%		82%	19%
					70.15%	99.58%	100.00%	52.59%	81%	23%
Mean	22.00%	29.00%	69.00%	64.00%	66.00%	98.00%	100.00%	53.00%	62%	28%
	71.25%	63.42%	100.00%	100.00%					84%	19%
		60.84%	100.00%	100.00%	100.00%				90%	20%
BCS			100.00%	97.68%	100.00%	50.18%			87%	25%
				90.18%	100.00%	51.06%	62.46%		76%	23%
					100.00%	49.15%	58.21%	58.75%	67%	23%
Mean	71.00%	62.00%	100.00%	97.00%	100.00%	50.00%	60.00%	59.00%	75%	21%
	100.00%	100.00%	2.85%	1.55%					74%	56%
		100.00%	40.76%	22.54%	100.00%				74%	40%
BOCLB			100.00%	47.86%	100.00%	85.54%			75%	25%
				100.00%	100.00%	86.13%	83.47%		74%	9%
					100.00%	86.53%	90.05%	60.98%	74%	17%
Mean	100.00%	100.00%	48.00%	43.00%	100.00%	86.00%	87.00%	61.00%	73%	24%
	100.00%	98.20%	100.00%	53.10%					88%	23%
		100.00%	100.00%	42.67%	82.59%				81%	27%
BIR			100.00%	43.29%	82.16%	59.60%			71%	25%

				43.88%	83.69%	59.62%	37.01%		56%	21%
					66.29%	53.84%	28.59%	38.71%	47%	17%
Mean	100.00%	99.00%	100.00%	46.00%	79.00%	58.00%	33.00%	39.00%	69%	29%
	99.17%	100.00%	39.63%	100.00%					85%	30%
		100.00%	46.35%	100.00%	75.71%				81%	25%
BCH			58.12%	100.00%	81.21%	58.58%			74%	20%
				100.00%	94.49%	79.36%	54.52%		82%	20%
					100.00%	84.70%	58.64%	61.54%	76%	20%
Mean	99.00%	100.00%	48.00%	100.00%	88.00%	74.00%	57.00%	62.00%	78%	21%
	100.00%	93.24%	100.00%	80.43%					72%	9%
		94.92%	100.00%	77.09%	100.00%				70%	11%
BNI			99.66%	74.17%	89.56%	100.00%			67%	12%
				74.14%	90.23%	100.00%	100.00%		65%	12%
					89.65%	100.00%	98.35%	100.00%	62%	5%
Mean	100.00%	94.00%	100.00%	76.00%	92.00%	100.00%	99.00%	100.00%	57%	8%
Mean	72%	69%	69%	65%	64%	61%	52%	53%		
St Dev	30%	29%	29%	29%	30%	28%	32%	27%		

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