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Should Economies Restructure towards Trade?: A Theory for Partial Specialization with Consumption Consistency

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Abstract: Why should countries trade the amounts, and kinds, of goods that they do, to maximize utility, and should this lead countries to orient their production completely to their trade sectors? This unorthodox, and primary conceptual article, proposes to answer whether or not completely specializing in goods, under Ricardian trade theory, is absolutely necessary. Many graphs are offered, with linear algebra, and equations, for countries to maximize utility. The main finding is that partial specialization can maximize countries' utility, negating the need to restructure solely for one good, industry, or sector. The implications for trade, using the United States as an example, are discussed. Several other original concepts are introduced, including the "box of surety," and the "ray of satisfaction," with the idea that trade should capture the same ratio of goods that consumers prefer domestically- "consumption consistency." Matrix algebra was used, along with general conceptual observations, with the inclusion of how math errors could be made, but future use of Calculus would add to the equational proofs, and, scholars should look for more empirical evidence of partially specialized trade occurring.

Keywords: *international trade; political-economy; restructuring; Ricardian theory; specialization*

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1. Introduction:

In 2023, many countries and firms, despite the global Covid-19 pandemic, which has affected supply chains, continue to restructure their economics towards their trade sectors, while other countries are turning inwards. Orthodox economic theory holds, in accordance with the works of early 19th Century British economist David Ricardo (1817), that countries specialize in the one good or industry that they are best at producing, relative to other goods. They then trade with other countries for the goods that the other country is relatively best in producing, for an overall gain. It is an economic theory of relativity, much like that of Einstein's theory for physics, called "comparative advantage" in economics, not to be confused with "competitive advantage," which is a strength that a company has such as a better human resources department than another firm. To put these ideas into "layperson's" terms for conceptual use, Einstein's theory in physics was, succinctly, that total energy is the largest mass times the speed of light squared, derived from the radioactive decay of carbon, that gravity bends light, and, relevant here, that time is relative to motion. If there were no motion in the universe, then there would be no time- nothing to compare by. Similarly, in economics, the utility (happiness) of each country depends on the productivity of the other.

N. Gregory Mankiw's (2015) microeconomics textbook has an unusual but modern way of explaining this, comparing the fictional movie character of Forest Gump, from the movie of the same name, to National Football League (NFL) quarterback Tom Brady. If Forest specializes in making money mowing lawns, and Tom Brady specializes in making money from football and advertisements, then they should each work at what they do best, and then trade what they produce, even if Tom Brady may be better at both, called "absolute advantage." But by dividing tasks, like with the division of labor theorized by Adam Smith (1776), they both end up gaining because the rate of exchange, for both, is greater than the relationship of their individual productivity for both occupations (Mankiw, 2015).

Another way of thinking about Ricardo's theories, added later in the late 19th Century neo-classical revolution, by British economist Alfred Marshall and also by the Austrian School of economists, is that countries completely specialize so as to not lose out in the other goods, which are called opportunity costs, and to gain more goods, thus increasing "utility," or happiness. With trade, states reach more goods, and a higher indifference curve of happiness, anywhere upon as such that the consumer is equally as happy between the two goods, as developed first by Edgeworth (1877).

These theories combined have been used by policy-makers to justify economic restructuring towards the trade sector, in favor of certain industries, whether government promoted, or based on entrepreneurial intuition, observation, research, or business deals/proposals. Is complete restructuring, which can cause the loss of countless jobs, and the closing of countless small firms in less productive industries, necessary, given Ricardo's models? And, how much should countries theoretically trade? The hypothesis of this article is that complete specialization is not necessary, using an addendum to Ricardo's theory and a methodology of several graphical models with matrix algebra, and that states maximize happiness by producing and trading to come close to their domestic consumption preference.

Ricardo's trade theory, and these works that followed, are based on the idea that the two countries can trade goods (barter) at a "terms of trade" agreement, which we know from subsequent works, can depend on: how much countries value goods (called "utility"); how large the country is in market power over a smaller country; the relationship between the values of currencies; tariffs; and the overall good's market, that is, if it were a monopolistic market, giving the monopoly state an advantage.

To continue with this brief intro, this terms of trade must be in-between the two countries' opportunity costs, which are more practically the slopes of their production curves, or trade will become unnecessary. However, Ricardo's theory, which has been questioned by scholars over the years for its vast amount of assumptions needing to make the theory true, but who have never found it to be false, unnecessarily posits that the new slope of the terms of trade must be extended from the point of complete specialization. The theory ignores that the new terms of trade can start along any point along the production graph, otherwise known as the production possibilities frontier, with a parallel line to the terms of trade. While this graph can be drawn as curved, due to increasing opportunity costs, or diminishing marginal productivity, they are drawn here according to the classical models, with straight slopes, for simplicity, in-line with Ricardo, and for making points without referring to Calculus.

It is important to note that most textbooks assume a starting point of consumption along the production curve without an explanation. Instead, this paper assumes, and explains, "consumption consistency," that, for instance, if one country can produce 3 oranges to 1 apple, that it makes sense that the country's citizens will consume 3 oranges to 1 apple; otherwise, why would a country have developed, over time, such a degree of production? Vice-versa, a country's production might have developed over many years in order to meet a countries' consumption preference. Which one of these occurrences that one believes in depends on how one views "Say's Law," that supply creates its own demand, which has long been held to be true in terms of primitive, less developed societies, while the opposite has been the case in more advanced societies that can cater to consumer demands. Regardless, consumption consistency is assumed for where one country starts consumption, and, that it should continue to where a country ends production through trade. If countries prefer a certain ratio of goods domestically, then why should they not prefer the same ratio throughout trade?

All of these assumptions are applied to both complete and partial specialization, with utility assumed to increase as countries trade for more goods, and combination of goods in-between consumption consistency for their two countries; that is, if consumption consistency for both countries is possible if the countries are of very different sizes. Complete specialization does still yield the greatest of all possible gain, as found, measured by the concepts termed here as the "box of surety," and the "ray of satisfaction," which is shown here as yielding the indifference curve which is the highest and is most consistent with consumption preferences of the home country. Nevertheless, potential gains can be made from the partial trading of the specialized goods, and still keep diverse sectors of the economy, and the associated jobs needed, used for producing the other good. Thus, complete restructuring of an economy towards specialization is not entirely necessary for gains in trade, and small businesses should be able to stay afloat in theory. All of these assumptions will be addressed as we go.

1.1. Literature Review:

There is much research on optimal tariffs and trade "policy," but very little on optimal trade "amounts." Most literature on trade involves those who expanded on comparative advantage, such as Hecksher and Ohlin (1919), who showed that countries specialize according to their resources (labor, capital, etc.). Meanwhile, Stolper and Samuelson (1941) showed that specialization causes "winners and losers" amongst industries that produce the resources. Krugman (1981) emphasized that multinational corporations do not always have to abide by the rules of countries, being situated in many countries.

To go back to the 1700s, Hume (1955 edition), before Ricardo, believed that trade benefitted smaller countries more than larger ones because they receive a greater proportion of goods to their size. Trade helped smaller nations develop with their standard of living, though for large nations, they gained less, but could not be hurt by trade, all called the "importance of being unimportant" (Pressman, 2014, 99). Edgeworth, in the mid-1800's, developed his famous utility "box" in which, Pressman writes, "[h]e discovered that there is not likely to be just one trading equilibrium point" (Pressman 2014, 99). The country that gained more had the better government which was more skilled at negotiating, that could "more easily do without the good produced by the other country," or had the most monopolies to charge what they wanted (Pressman 2014, 99). Much depended on the number of firms in each country; countries with many firms, mainly in large nations, would see greater competition, and its firms would benefit less from trade (Pressman 2014, 99). Many authors have written on the "winners and losers" from trade.

Ricardian trade theory is largely a model in favor of free trade, because tariffs raise prices, distort the terms of trade, and favor certain industries over others. Recent work on international trade includes Maneschi (1992) finds empirically that "foreign trade only affects the rate of profit [not productivity] insofar as it leads to the importation of cheaper wage goods." Siddiqui (NA) writes that historically, productivity [and likely utility, too] under free trade has not risen, and "free trade policy will deepen further the process of uneven development and unequal exchange" across the world, due to the "weak theoretical and empirical grounds" of free trade. In doing so, the "performance of both sectors [industrial and agricultural] could [both] have a long-term impact on … well-being." A number of articles within the past several decades have questioned the extent of globalization that the world has moved to. Many books claim that Covid-19 and limited travel (with bans), backed-up ports, and supply chain precautions, have reduced globalism.

Levchenko and Zhang (2014) write that, in an analysis of 72 countries over almost 50 years, "comparative advantage" has been weaker than expected. Trade productivity from technology, as well as the amount of trade itself, has not grown as much as it should have under Ricardian theory. Seretis and Tsaliki (2015) find that countries maintaining absolute advantages in trade (producing both goods at a more productive rate) tend to hold on to these advantages rather than switch to relatively better gains in comparative advantage, in four Euro-zone states, since 1995. The problem, writes Alan Deardorff (1984) is that "the model (Ricardo's) implies complete specialization in equilibrium… since imported goods will almost never be produced in the importing country." Such models "explain only the direction of trade, not the quantity of trade" (Deardorff 1984, 473). This is the conceptual goal of this paper. Finally, "partial specialization" appears in newer texts, but it refers to productivity losses in neo-classical models that cause countries to lose comparative advantage, whereas here, comparative advantage does not change. Finally, Peet and Hartwick (2015) critisize Ricardo for not stressing that some specializations arose by goods being backed by wealthy European monarchs, thus leading to income inequality.

2. Materials and Methods:

The methodology of this article will be to first address the many additional assumptions, to those touched upon already, that Ricardian trade makes, and also the additional assumptions of this article. The article then presents the theory of comparative advantage graphically, and introduces such new concepts as complete specialization, the box of surety, partial specialization, uneven gains, the ray of satisfaction, and partial specialization with the ray of satisfaction. The article builds to the concept of "consumption consistency," which the country starts out with in term of its level of production, and then should continue with trade. This concept was also mentioned in the introduction.

The paper will then apply these concepts to an example using two countries of A and B. It then introduces basic math to find the quantities that balance trade for partial specialization, in the case that countries wish to produce numerous goods rather than entirely those that it is best at. Partial specialization is shown as helpful in reaching final consumption points near the original "rays of satisfaction," thus obtaining "consumption consistency," especially for countries of unequal size. Several more sections and equations are then offered, for finding what are called "exact," "average," "doubled," "balanced," "singular," and "extreme" rays of satisfaction, with "shifts," and "intersections," for partial or complete specialization, to maximize utility. Assumptions are made for productivity losses, and for trading outside of the "boxes of surety." A series of equational steps is then shown to maximize utility, followed by an overall list of steps in whether a country chooses to use complete or partial specialization. This is followed by an extra example for medium-sized countries. Finally, a discussion rolls forth with brief empirical notes about partial specialization and possible countries' losses from trying to restructure to a sole, complete specialization model. Countries can still maintain smaller industries. This part touches on several applied cases of countries and firms, with a section suggesting how future research can build upon the work presented here, and a conclusion summarizing the findings. All of the math work is matrix/linear algebra with simultaneous equations.

2.1. Ricardian Assumptions:

Ricardian theory depends on many assumptions, which will be outlined here, and many will still be used in regards to the theory presented, as well as with the additional, new assumptions of consumption consistency, towards the theory of partial specialization. Ricardian theory, to begin with, assumes a fixed population of labor for all production, and that labor never declines due to frictional, structural, or cyclical unemployment, which would result in a lesser production of one good, or all goods. Furthermore, it assumes that all workers have the skills to switch to complete specialization, such that countries can always produce more goods to trade ("Ricardian Model Assumptions," NA, 1).

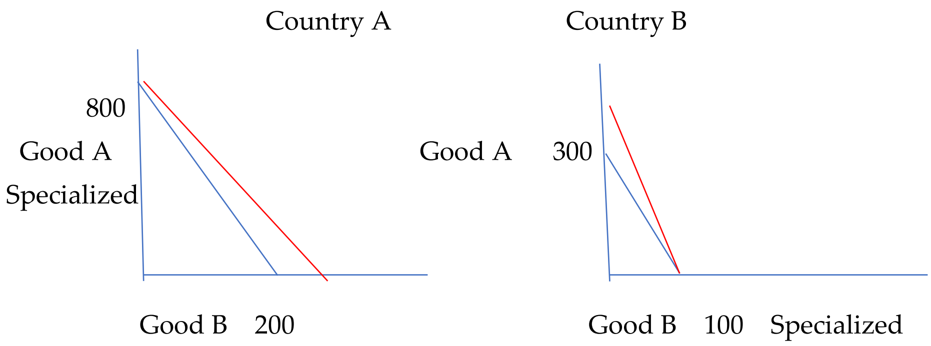
Labor is usually the main resource or input in Ricardian models, and not capital, because capital would involve depreciation, as well as diminishing returns, and curvilinear graphs. The theory also assumes that labor by skill is static and that workers do not move to the countries where their skill sets would be more in-line with the goods produced there, which would impact the quantities produced. Another Ricardian assumption is that perfect competition exists domestically in all factor markets, such that one can always obtain the other resources to produce as many goods as necessary. And, there is no government intervention, an assumption challenged here, and by others, as we know states often help sectors. But specialization, natural or government-led, comes about through, and causes, restructuring.

Ricardian theory also assumes that there are no tariffs, which could affect the terms of trade, as an added price. Further unrealistic is the assumption that trade occurs by barter. In such models, trade could also be expressed with money values, but in exchanging goods, again, trade would depend on the cost of production in the terms of trade- thus, supply. There is nothing else to go by, such as demand. Demand depends on tastes and preferences, which would affect the terms of trade, in how much of the other good one would want, but these tastes are assumed to be the same over time, and typically, there are only two goods assumed to be in the market. It might be possible, though, through very advanced Calculus, to expand to more dimensions of many more goods, with many more gains from trade.

Furthermore, Ricardian models assume that production is efficient with resources, and that production usually falls along the lines of the production curves (PPF's, Production Possibility Frontiers). There are no presumable shifts outward of the curves due to more capital or technological productivity over time (Siddiqui, NA, 1-2). Moreover, the model assumes no transportation costs to ship the products to the other country, which Ricci (1997) found affected the magnitude of trade, though this cost might be included in the terms of trade. While Ricardo did indeed foresee production possibilities with risk, such as if one country had a "shock" or natural disaster, he did not include this possibility into his model of comparative advantage, and it is left out of the models here, thought this idea is mentioned. Particularly relevant to this article, the work assumes that domestic preferences will stay the same with trade- that consumer's preferences will not change over time. With preferences, the paper also assumes that domestic production is in-line with domestic consumption, or the reverse- "consumption consistency"- and that this preference continues throughout trade. Finally, the model is pre-neoclassical- it is simplified and does not account for diminishing returns to scale, which would make the lines curved and more complex to calculate, which is left for others.

Figure 1

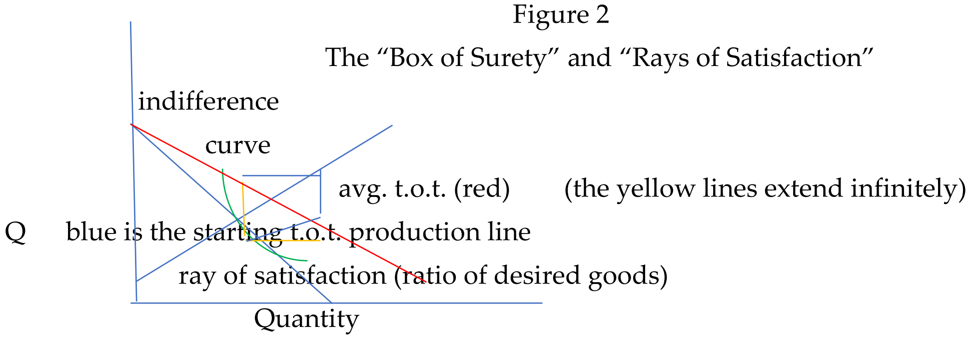
Complete Specialization



Here, we build to the final theory with original graphs. The opening up of trade (in red) gives the countries involved a chance to consume at a different rate, or slope, in the trade-off between goods, which is the terms of trade line, and is in-between the two countries opportunity costs (1/4 and 1/3) of the comparatively advantaged goods. Otherwise, the purpose of trade in at least one country would be obviated and production and consumption would continue unchanged. Pre-neo-classical times, opportunity costs stay constant. This 3.5 terms of trade (t.o.t.) is the red line above (Figure 1). Complete specialization would mean making 800 and 100 goods, respectively, for countries A and B.

Figure 2

The "Box of Surety" and "Rays of Satisfaction"

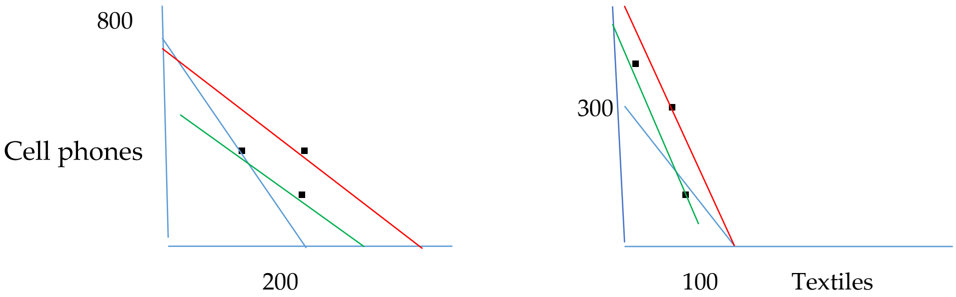


Countries are assumed to continue to consume the same ratio, where the ray of satisfaction (a preference ratio), called by Schotter (2009, 68) the "Income Expansion Path," but hereafter simply called "rays," intersect the red production line, which is consumption after trade. Although it is not absolutely necessary to assume consumption consistency in order to create the theory of partial specialization, this assumption nonetheless helps to develop a closer-to-accurate model, and later is used in the examples to better illustrate the concept of finding maximization for partially specializing. The green "indifference curve" in Figure 2 comes from Edgeworth and indicates that points on the curve, which are tangent to the production lines, represent equal utility to the consumer. To ensure that the indifference curve has increased (equal satisfaction with the two goods), the curve either must increase vertically, to gain more of one of good, horizontally, to gain more of the other good. Or, it must have increased in some combination of the yellow box (the box of "surety") (Figure 2 and Figure 4), beyond the original point of domestic production and consumption. Here, one can be fairly sure happiness is stable: there is either a gain in one good with no loss to the other (for either good), or some mixture inside the box.

Utility gains outside of the "box of surety" are still possible, but not guaranteed. If, in the case of the downward production slope is steeper than 1, then a country will lose utility as the box extends rightward, but gain if the slope is less steep than 1, with steepness being known as "elasticity." Even so, one cannot be sure about maximizing utility, because all one has to go by is the total quantity of goods, and not the mix, and one is assuming that consumption in a country can be aggregated when consumers have different preferences, as questioned by Arrow (1951). With trade, the terms of trade (t.o.t.) causes a movement towards the less specialized good, such as from a bad trade deal. In this case, the ray of satisfaction continues at a rate unchanged, all of the way to the right until an infinitely beneficial terms of trade is reached. At this point, the country has maintained the same amount of the specialized good, and increased substantially the unspecialized good traded for. Here, "t.o.t." are an average: 3.5.

Figure 3

Partial Specialization

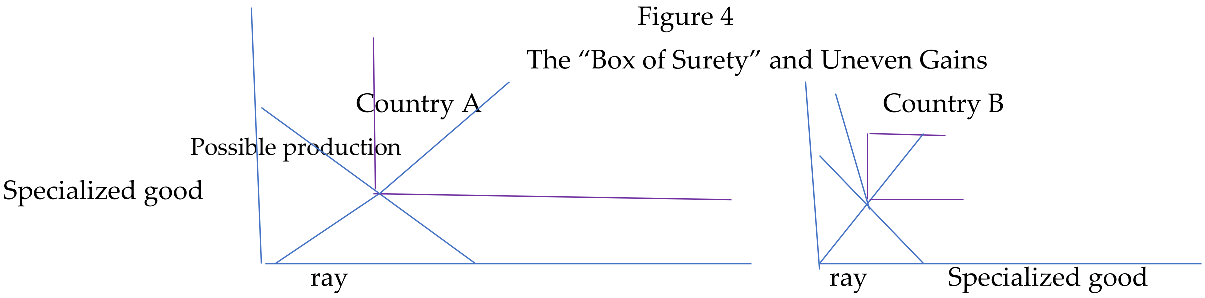


The country on the left has the comparative advantage in producing cell phones, since it gives up ¼ textiles, while the country on the right has the comparative advantage in producing textiles, since it gives up more, 1/3, to produce cell phones. One can see, on the left (Figure 3), that partial specialization (given in green) results in a parallel terms of trade line, but from the point of production nearer to the domestic economy, and not from complete specialization at a point of 800 cell phones. There still is an increase in the number of goods that are able to be consumed (textiles), although the number of cell phones has gone down. Whether or not there are more textiles than under complete specialization depends on the specific details of the problem, that is, the terms of trade and the amount of goods traded, which will be addressed later. And, of course, we are assuming that the terms of trade do not change drastically whether a country uses complete or partial specializing, so trade is still acceptable to both countries, and still balances. Regardless, there is a gain in the goods which the country is lacking in (textiles), so if there is a strong demand for these goods, then, despite the loss in cell phones, partial specialization has still fulfilled its purpose. The country gains without having to completely restructure its economy, through business deals or state policy, towards complete specialization.

To reduce risk, the country can also diversify its production. Diversification prevents against calamities that could be associated with producing only one type of good. This echoes the findings of Barry (2020) and others, who, interestingly, found that larger nations tend to be the ones able to take on the risks of specializing. Smaller GDP countries are more prevalently tending towards diversification.

Figure 4

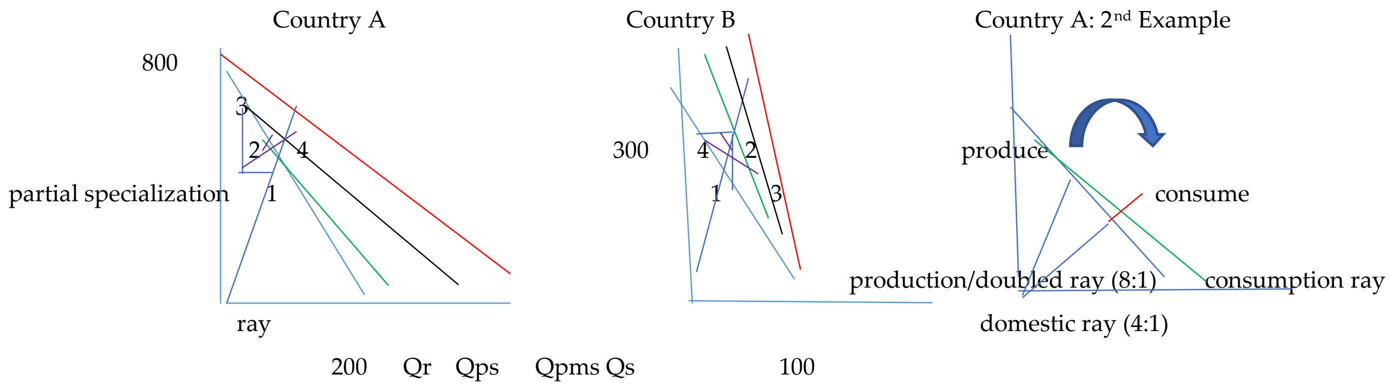
The "Box of Surety" and Uneven Gains



Trade usually causes a gain, and an overall increase in production, more-so to the specialized goods, which are the most traded. It is assumed here, though, that the original ratio of goods consumed continues. Trade results in consumption that is consistent with *domestic* consumers. This ratio is represented by the ray of satisfaction, if one were to extend it from the origin (Figures 4 and 5).

Figure 5

Partial Specialization (more shifted) with the Ray of Satisfaction



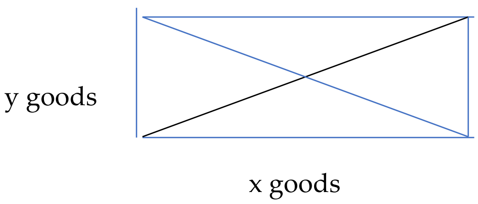
Qr= quantity regular, Qps= partially shifted, Qpms= partially more shifted, Qs= specialized

Most textbooks on international trade simply assume the starting point of domestic production and consumption, as well as the quantity of the amount of goods traded (although they are indeed at a specific ratio: the terms of trade). They also assume the terms of trade, themselves, so long as the terms of trade are between the two opportunity costs. Similarly, this paper will also assume the terms of trade, though it is likely that the larger country can assert more power over it. Also, complete specializing may result in more goods after trade necessary to satisfy domestic customers. Partial specialization is better geared towards meeting consumer's needs. But, this does not always occur for both countries at the same time. In line with Barry (2018), this section of the paper makes a case that countries will attempt to trade the number of products closest to maintaining the same utility relationship in domestic consumption (and adjust production), that exists prior to trade- *"consumption consistency."* This, one should assume, should render a finish point after trade which is close to domestic production, and is not just randomly assumed. One consumes more of the good that you can produce domestically, and less of the one that you cannot, with trade resulting in *more* goods overall.

In Figure 5, if a country truly desires a good, it will sacrifice its terms of trade, and greater quantities of goods, to maintain the right mix. To do so, however, trade (whether complete or partial) will result in more of the non-specialized good. Therefore, domestic production (Qms- "more shifted") of the country in partially specialized trade must, in the left-hand of Figure 5, shift away from consumption and slightly closer towards the specialized good, to trade. Thus, on the left graph, domestic production moves from point 1, to point 2 (the box is not shown), which is bypassed (its box-of-surety birfucating ray does not connect with the original ray) because the terms of trade would yield less of the desirable good. This results in production at point 3, which, with the new, bifurcating ray at 45 degrees (the mid-point, assuming the two goods maximize utility in trade; see Figure 6). Final consumption occurs at point 4, consistent with domestic production. Through the concentric triangles (or "boxes of surety"), the same mix of goods is maintained. However, on the far right, a more simple view is shown: consumption consistency is from the original domestic production ray, and final consumption extends from here (in red). This fewer-step-method is this paper's approach. One can see on the left-hand graph that the movement from trade is almost always down, towards the non-specialized good, while with the middle graph, the movement is usually upwards, towards the non-specialized good.

Figure 6

Bifurcation Maximizes Utility



In leading up to the examples, the final, basic graph (Figure 6), shows that a ray of satisfaction which bifurcates the production line leads to the greatest area underneath it. This ray yields utility maximization, which will be discussed more later on. If the production line were steeper, then a ray with greater upward slope would be used, and with a steeper production line were flatter, than a flatter ray would be used to maximize utility, which will we see in examples later in this paper. Now, we turn to examples, showing that complete specialization might be unnecessary to maximize utility, if a country chooses to partially specialize. Finding the right (utility-maximizing) ray is essential to determine the finishing points after trade, and therefore the trade amounts themselves, but trade must still balance, as the finish points must be on the curves where the rays intersect the production lines (also called curves).

2.2. Examples Using Matrix Algebra:

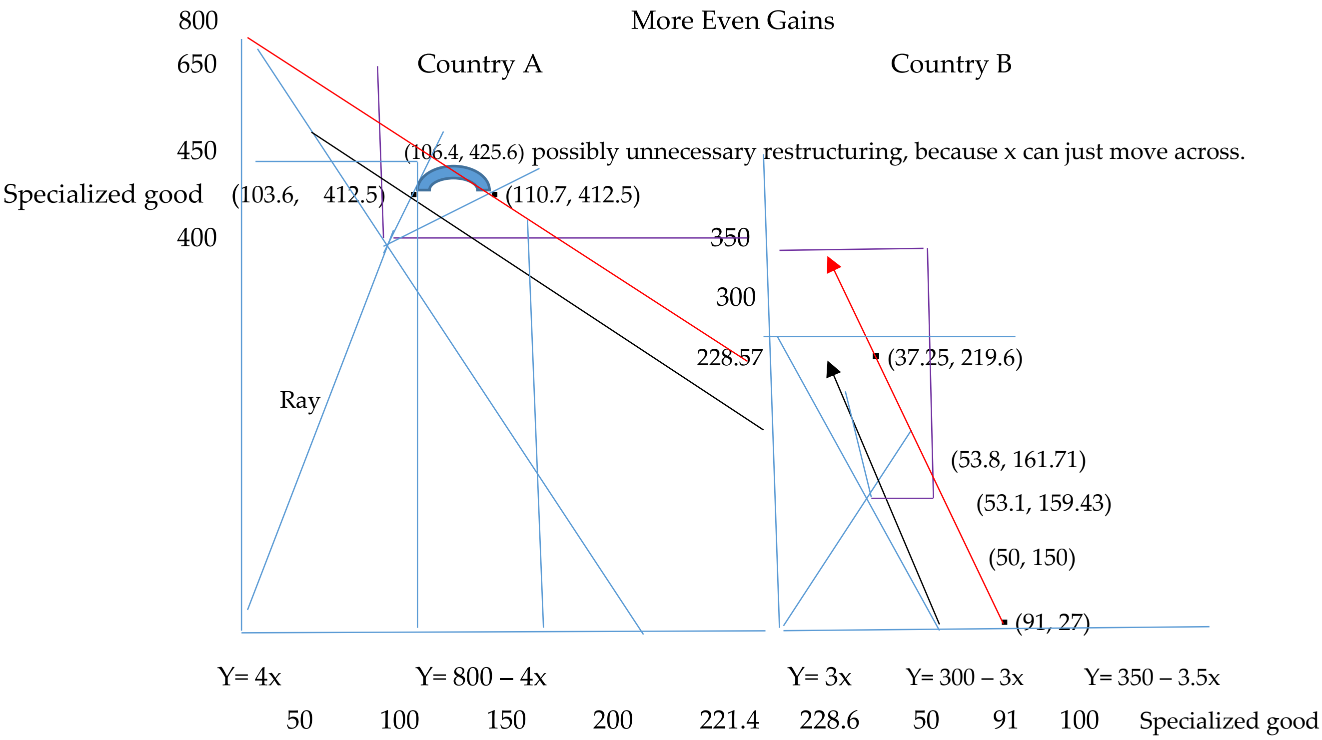
The works of Salvatore (1996), Grieco and Ikenberry (2003), Sawyer and Sprinkle (2009), and Kreinin (2010), all assume, with no ostensible reason given, the amount of goods traded and the terms of trade. The following identities (Equations) in 6b can be used to determine the amount of goods traded by both countries in order to maintain Country A's consumer preference ratio, which, in this example is 4:1. The same ratio must be used initially in Country B's equations; more will be explained. Thus, while some countries influence the terms of trade by the situations already discussed, they may also influence the amount of goods traded, such that the consumers in only one country's happiness is "consumption consistent." However, striving for this ratio could possibly mean less overall happiness for that country; in fact, for one country aiming for a consistent ratio, with partial specialization, the country usually ends up with lesser goods and a lower indifference curve than under complete specialization. However, it obviates the need for restructuring, and it can render the other country better off in their ratio and in their indifference curve (number of goods). We will see if both countries can obtain a "consumption-consistent" ratio at the same time. One state's trade influences the others'.

3. Results:

In Figure 7 below, each country is limited in goods by the amount of production of the other country, the "limits" of which are represented by the completely horizontal and vertical blue lines, which textbooks do not mention. But, by starting at a non-specialized point, countries can exceed these limits. A starting point must be assumed, but with reasons- consumption consistency, continued through the red lines. If Country A produces 600 specialized goods and trades for 53.75 of B's specialized goods, assuming Country A and Country B are using partially more shifted production (see Figure, 5), at a terms of trade of 3.5, which is the average of the two slopes (the red and black lines), the 4:1 ratio is maintained (412.5/103.6). Further restructuring for Country A may be unnecessary, due to the costs to society. Unemployed workers can yield less output, another reason why restructuring may be harmful to an economy. What happens if one country decides not to restructure and what effect would this have on the other country? Certainly the situation would not be good, as one country would be left without a trading partner, unwilling to trade excess goods. With partial specialization, the smaller country can satisfy the larger one, and both countries come closer to their rays of satisfaction.

Figure 7 (not to exact scale)

More Even Gains



3.1. Initial Equations- Partial Specialization Starting with Country A:

The following equations show, roughly, how a country can pinpoint an ending point, after trade, and therefore how much to produce before trade, using partial specialization. In Figure 7 (above), the blue line rays of satisfaction are before trade. The red production possibility frontiers are consumption after production have shifted towards trade, then after trade has occurred, and the black production possibility frontiers (partially-specialized), starting before after production has shifted, and end "after trade." Country A produces at the maximum ("end") points of the y and x graphs (800y and 300x with complete specialization, and at the intersection of the black and red lines with partial specializing) before trading. The small countries production must change from a constant of 300 to a constant of 350 with specialization. But, because A starts at a certain chosen point, then the percent of goods that Country A decides to "shift" in not producing specialized goods affects the percentage "shift" in Country B. This results in both being able to obtain more non-specialized goods. Since Country A "shifts" 3.125 % ((800-775)/800), then Country B might shift as well, but this is pinpointed later on in the paper. Below, when there are spaces with x, x is a variable, and when there are not, x represents multiplication.

Equilibrium for partial specialization given country A's chosen starting point (based on how much it decides to restructure towards the "end," or maximum amounts, before trading):

Slopes, or rays, of 4 and 3:

Country A and B: <u>y2- y1</u>= slope, which are more properly described at the bottom of Fig. 6.
x2- x1

1a]

<u>Countrya:ystart - Countryb:ytraded x terms of trade</u>=	<u>Countryb:xtraded x terms of trade + Countryb:ystart =</u> <u>4</u>
Countrya:xstart + Countryb:xtraded	Countrya:xtraded (from left-hand equations)

Countryb:ytraded must be > than Countrya:start

Countrya:xstart + Countryb:xtraded must be > that Countrya:xstart

3.2. Initial Equations- Starting with Country B:

1b]

<u>CountryBystart + xtraded x terms of trade</u> =	<u>CountryAystart - CountryByfinish</u> = <u>3</u>
CountryBxstart – CountryBxtraded	y traded [or CountryAxstart + xtraded)

Countryb:ytraded must be > than Countrya:start

Countrya:xstart + Countryb:xtraded must be > that Countrya:xstart

Given the terms of trade: the lesser the terms of trade, the greater the utility gain for the smaller country, with the steeper production line, which will also be shown throughout this article.

3.3. Partial Specialization Math Example, for Country A:

Proof of Theorem for Equations 1a] and 1b]

<u>Country A</u>: given (50, 600): From restructuring: the black line, to reach equilibrium.
<u>600- y(given) x 3.5)</u> = 412/103.75= 4 and from simple restructuring (moving along the blue *crescent*), 50 + y or "arc," rather than having to completely reorient its economy to the specialized good: y = 800 – (3.5 x 103.6) = 110.7 x

<u>Country B</u> When x = 100, y = 0, so y= 350 – 3.5x
<u>xtraded x 3.5 + y start (27 solved)</u> = 4 (the slope of the ray of satisfaction for Country A) x traded (53.7)
<u>53.75 (3.5) + y</u> = 4 (the slope of the ray of satisfaction for Country A) 53.7 y = 27
27= 300 – 3x, x = 91 After trading: x= 91- 53.35 = 37.25 y = 350- (37.25)3.5 = 219.6

OR, choosing any starting point:

With an x or abbreviated (partially specialized) y start:

For Country A: [y starting point + 3.5 times x start] = 775

With a y start, using a matrix:

Simultaneous equations at black line domestic production:

y= 775 (calculated) – 3.5x

y= 4x

For Country B: with an abbreviated (partially specialized) x start:

x starting point (assumed to be 0 here) + (y start/3.5) x times 3.5= (y maximum/end – shift)

Simultaneous equations at black line domestic production:

Y- (y maximum/end – shift) – 3.5x

y = 3x

3.4. Partial Specialization Math Example, for Country B:

-continued

$$\frac{27 + (37.9 \times 3.5)}{91 - 37.9} = \frac{(600\text{-}219.6\text{-}27)\text{ (calculated)} - 132.65}{37.9 \times 3.5} = 3$$

There is statistical discrepancy from rounding, and from the equations needing more refinement and less estimation, which will be shown later. Throughout this paper, we will look for more elegant and convenient ways of using math so that countries' finish points (after trade) come closer to their starting points (before trade) at their initial "rays of satisfaction" for "consumption consistency." Again, the *blue crescent* (or "arc") for Country A in Figure 7 simply compares points with greater specialization to points from partial specialization. It may not be necessary for Country A's economy to completely restructure towards the y good before trading, as it can use partial specialization, and then simply expand production of its x good slightly to obtain the desired ray of satisfaction.

3.5. Further Results- Utility Maximization:

We now attempt to find the utility maximization for rays of satisfaction with trade that benefits both countries, and is close to their initial rays of satisfaction. Again, this is so as to maintain consumption consistency, and it is for either partial or complete specialization: the two types of trade are addressed together in this section, for brevity, as the assumptions are the same for each.

A general observation is that a one-country shift in the starting ray of satisfaction will double the other country's ending ray of satisfaction, because the other country needs to supply both countries with enough goods. Thus, the first step in maximizing utility, if individual "exact" rays are not possible (4 and 3), which they are not, using the math, then one's next step should be to look for such a "doubled" ray, which is the smaller country's ray doubled in order to satisfy itself as well as the other country.

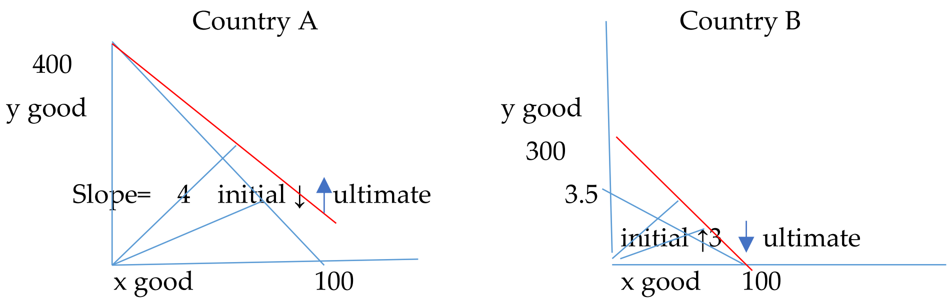
In the previous example, in trying to solve for Country A's "exact" ray, an initial slope of 4 had to be used for both countries, to start. This was to ensure that the ending points fell on the graphs for both countries, simultaneously, with the desired amount of trade. When starting with Country B, we used the number 3 in the example, for the slope of Country B's ray. In this case, trade more greatly benefitted this country. The ray could not have been 3.5, due to the "limits." Country B is too small to satisfy Country A. Also, the "exact" rays cannot work for both countries simultaneously with complete specialization. Thus, one would turn to a "doubled" ray, followed later on by seeking an "average" ray. Points closest to the rays of satisfaction, as we will see, are usually "pulled" by the larger country (see the directional lines). An "average" ray of [3 (initial slope of A + initial slope of B)/2, in this case (4+3)/2 = 3.5 of the ray values, could be taken between the two countries. This ray would increase utility (see the math later on), other than with using the exact rays themselves. Still, the rays must yield points which fall on the production curves with the desired, balancing amounts of trade.

Consider a simpler example, using Figure 8, which has the same slopes as before. Country A is smaller, so Country B is indeed able to completely satisfy it through trade. While 3.5 of a slope of both rays of satisfaction would be the maximum utility (shown further ahead with math), the "exact" values for x and y on the red lines are not possible through trade. Again, the countries are different sizes, and need a "doubled" ray. Countries, though, might still trade to be *near* these spots if they are willing to sacrifice some goods. Countries close in size will trade better than those that greatly differ, because the trade will better match consumer preferences. This is conceivably why large nations like the United States, China, Germany, and Japan trade heavily with each other today, rather than more-so with smaller states, and smaller, developing states on the whole tend to trade more amongst themselves.

3.6. The Ray of Satisfaction Used in Production:

In this smaller example, the exact rays: (46.15, 138.9) (68.5, 224), and "doubled" rays: (32.5, 189.5) (68.5, 274) do not allow for the desired, balanced amount of trade for both countries.

Figure 8 (calculating averaged rays of satisfaction)



In Figure 8, maximization of utility occurs at the "exact" rays where they are 4 and 3 for countries A and B, but this is not possible with complete specialization. A "doubled" ray also does not fit on the curve. These are both determined with simultaneous equations. It is hypothesized from conceptual, mathematical observation that the next possible maximization between the points of 4 and 3 occurs at (4+3)/2 = 3.5, an "average" ray. By not meeting the "exact" rays, the larger country and smaller country both may lose domestic consumers by producing more goods that they do not prefer. But, the goal, specified earlier on, is to come as close as possible with the rays to domestic consumption. And, the "average" ray is the same slope as the terms of trade. So, the average ray bifurcates the new, red curve. For this next option of using "average" rays, total utility is increased. Both countries "initially" try to move to this 3.5 average ray. However, instead, they gravitate towards their initial rays, because these rays are "exact." Still, as the average ray becomes smaller, the larger nation gains more, but as the ray becomes larger, the small nation gains more, shown in the math work that follows, in 6h.

In the math work below, maximization of utility is assumed to be x times y, as according to Edgeworth, the area beneath a curve. We search for a ray between 3 and 4 that would balance overall utility. Utility is usually assumed to be measured by the area within the indifference curves. But, the slope of these curves is unknown here, and making an assumption would involve more complex Calculus that is beyond the scope of this research. The totaled numbers below are farther away from the small country's starting utility, meaning that it has benefitted more proportionately. Yet, both countries benefit from trade reaching a higher indifference curve, whereby there are more goods.

The math that follows first shows utility being calculated, and then shows that because 3.5 does not fall on both countries' red line production curves simultaneously. Therefore, it is necessary to find a "balanced" ray of satisfaction, which results in rays of 3.73 and 3.27. This math generally corresponds with the math that follows using equation [4a and [4b for the larger country example. These points yield the next most utility, barring any losses from moving out of the "box of surety." Such losses are shown later in Table 1. Potentially, indifference curves may fall outside of the "boxes of surety," which requires more assumptions, shown later on. Using the outer rays of 4 and 3, or 3 and 4, for either country, does not yield higher utility than 3.5. At some point of applying the rays, only losses would occur, because the points would be outside of the "boxes of surety," or, outside the maximum of goods' "limits," unless these assumptions were set aside. As already stated, if countries produce in expectation of trading, and less, or no, trading occurs, then there will be a loss to the utility of domestic consumers.

3.7. Maximizing Utility, using the same, small countries example:

Proof of Theorem of Average Rays

Start of production: (with the numbers all roughly rounded)

Country A

Rays of Satisfaction (added vertically)

4	3.5	3
Y = 800 – 3.5x		
Y = 4x	y = 3.5x	y = 3x
7.5 x = 800	7x =800	6.5x = 800
y = 4x		
x time y for the area of the box;		
106.4 x 425.6	114.28 x 400	123.07 x 369.2
+	+	+

Country B		
Rays of Satisfaction (added vertically)		
3	3.5	4
Y = 350 -3.5x		
Y = 3x	y = 3.5x	y = 4x
6.5x = 350	7x= 350	7.5 x = 350
Y = 3x		
x times y for the area of the box		
53.8 x 161.71	50 x 175	46.67 x 186.67

Utility = 53,983.8 total 54, 462 (ideal maximization) * 54, 152.1 ▫

*Utility is greatest at 3.5, the midpoint, which seems always to be true (see Figure 6), and it is an important point in this paper. But, since the points for the 3.5 average ray do not fall on the red slopes for both curves at the same time, then what we can call a *"balanced ray,"* shown mathematically *below*, needs to be calculated now to maximize countries' utilities. This is a ray which comes close to the averages. It falls on the curve if the countries are similar enough in size, and maximizes utility for both countries, and thus both combined.

3.8. Finding "Balanced" Rays with Matrix Algebra:

Let us turn back to Figure 8 earlier. To reach the red lines, C and D are the slopes of the rays of satisfaction at maximum utility, where trade is possible. Theoretically, the countries should come close to maximizing utility at points with "balanced" rays of satisfaction, which is further tested ahead. To therefore solve for C and D (the "balanced" rays), we can use the following equations:

Proof of Theorem of Average Rays with a Small Country

Country A y = 400 – 3.5x x = Dx
Country B y = 350 – 3.5x x = Cx
All ending points should fall on the red lines, and production lines, so: Dx = 400y Cx = 350y Thus, D = 1.14 C

Using the theory of an average ray:

(C + D)/2 = 3.5

C + 4/3.5 C = 7

2.14 C = 7

C= 3.27

D + 3.5/4 D = 7

D = 3.73

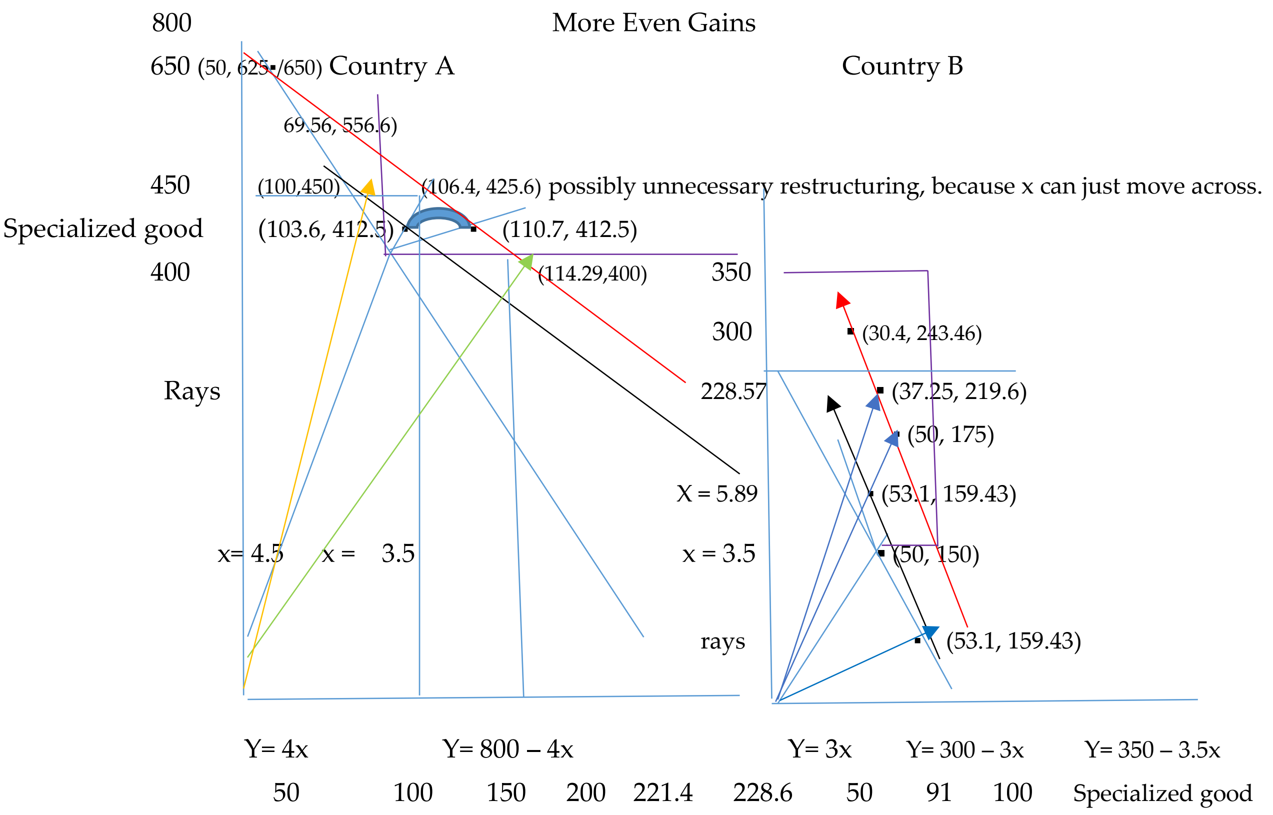
The values above fall on the curve:

(59, 192.93) - The large country: gains 59x, sells 206y

(41, 156.8) - The small country: produces 100x, sells 59 for 206y.

3.9. Finding "Balanced" Rays with Figure 7, and Figure 9 Below:

Figure 9 (not to exact scale)



3.10. Figure 7 and Figure 9- "Average" and "Balanced" Rays Math:

In Figure 7 earlier, and Figure 9 directly above (which shows all of the points we have, and will, discuss) with the 3.5 slope of the terms of trade, "exact" and "doubled" rays are not possible. Then, a bifurcation, with an "average" ray of satisfaction of 3.5, will give the greatest overall utility. This is in keeping with the original domestic preferences and staying within the "boxes of surety." And, the points derived must fall on the red lines of trade. If one chooses a different starting point (with partial specialization), using the black line, then these points will be different and a different ray of satisfaction might be needed. This is needed in order to have ending points that are possible with the starting amounts. As stated, "exact" and "doubled" rays are not possible, and an average, 3.5 ray, being the "average" between the original two rays of satisfaction, does not yield points which fall on both curves simultaneously. Then, a "balanced" ray would be the next step in providing each country with maximized utility, along with the example presented with Figure 8. Via these line, if this does not yield points on the curves, then proceed (see Table 4 later on) to find a *"singular" ray*, discussed next.

Before finding a "singular" ray, the math for a "balanced" ray for Figures 7 and 9 is shown below. Since the balanced ray is not possible, then ultimately take "singular" rays (see ahead). Or, if not possible, then "extreme" rays might be necessary, which are shown by the yellow and green lines within Country A that the small country, Country B, must meet in order to satisfy it, in Figure 9. Exact rays with slopes of 3.5 do not fall on both curves concurrently, so, as stated, first look for a "balanced" ray:

Proof of Theorem of Balanced Rays

Y= 800 – 4x Y= 3.5 x ray of satisfaction
Y= 300- 3x Y = 3.5x
Using simultaneous equations: x= 114.29, y = 400 x= 46.15, y= 161.54

For Figure 7, and Figure 9, using the math below, a "balance" of the rays would result is a ray of 4.87 and 2.13, but the smaller country cannot provide the larger country enough x goods, without "abbreviated starts"- that is, assuming a starting point closer to the center of the graph, explained further in section 3.15.

These balanced rays can be determined by the following equation, which is derived from the math work in this "Finding Balanced Rays" section, where t.o.t. is, again, the terms of trade:

4a] Country A ray = (A t.o.t. + B t.o.t.)/ [(slope A/slope B) + 1] = 7 / [(800/350) + 1] = 2.13

4b] Country B ray = (B t.o.t. + A t.o.t.)/ [(slope B/slope A) + 1] = 7/(350/800) + 1] = 4.87

These equations consider only the slopes of the original production equations, and not the constants in the production equations. Then, the resulting slopes for the utility-maximizing rays of satisfaction do not fall on the original curves, since the smaller country cannot satisfy the larger. Via the observations made here, the next step should be to find an "equal," or here-called, "singular" ray. This step would be to seek points that fit on the production curves of both countries, to maximize utility.

3.11. Finding "Singular" Rays:

Figure 10 that follows shows utility based on x times y. To maximize overall utility at this point, from observation, it becomes necessary to use "singular" rays, which will be shown with math that follows about maximization. In calculating these rays, they do not always fall directly between the two countries initial rays of satisfaction, because the trading countries are different sizes.

Using Figure 7 (and later Figure 9) to obtain the same ray for both graphs, as an example, then they balance at a ray of 8, and for Figure 8 earlier, at a ray of 4. Both of these numbers can be obtained by dividing the outermost y value for the larger country by the outermost value of x for the smaller country, or: 800/100= 8, and 400/100= 4. Therefore, to find a "singular" ray:

5] The equation here is: (total productivity of the larger country y/ total production of small country x).

For the value 8, this results in the points of (69.456, 556.54) and (30.44, 243.46) in Fig. 7. The math showing the more elegant derivation for the 4 unit is shown below:

Proof of Theorem for Equations 4] and 5]

Bx = 100 – Ax

400/ (3.5+ray) = Ax

350/(3.5+ray)= Bx= 1- Ax

400 = 3.5 Ax + Axray

350= 350 – 3.5Ax + 100ray- Axray

Adding these two equations

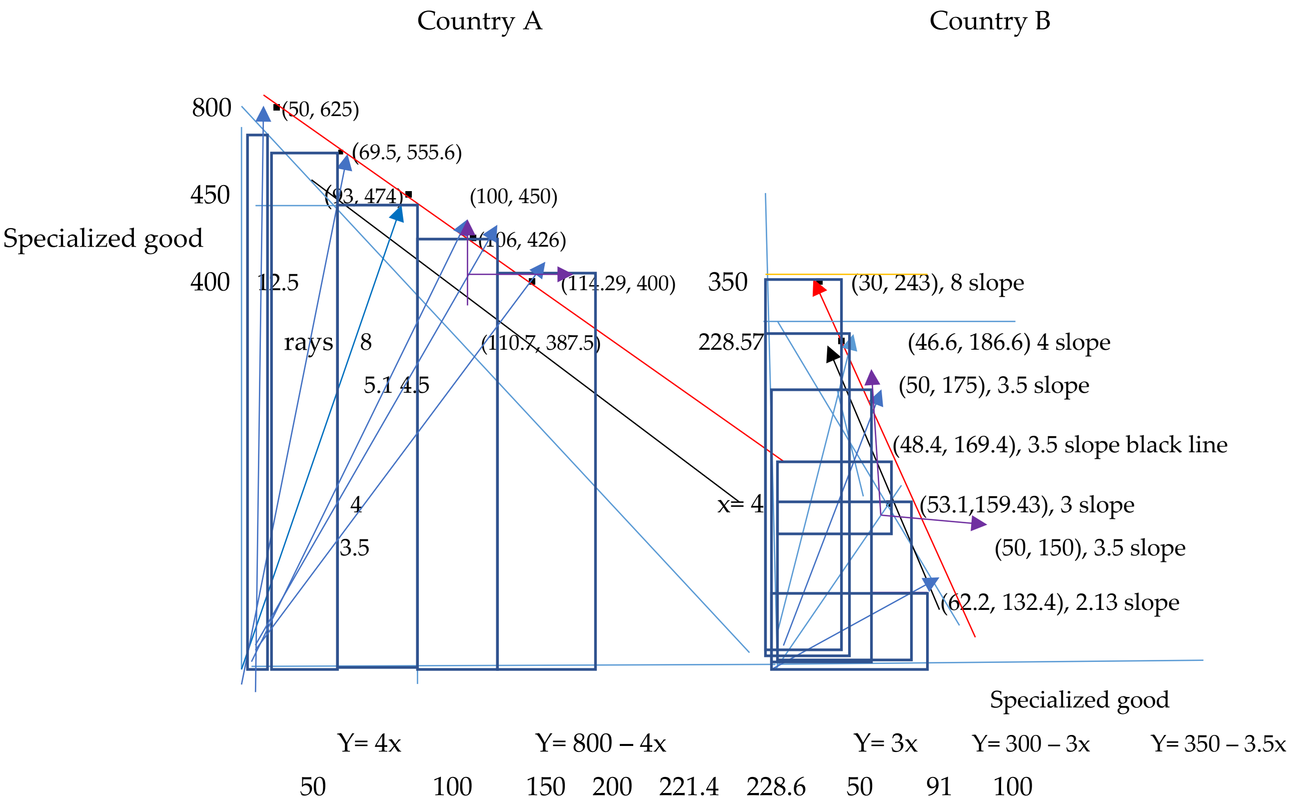
3.5Ax – Axray – 400 = -3.5A + 100 ray – Ax ray
400 = 100ray

ray= 4 ▫

3.12. Utility with Various Rays:

Figure 10 (slightly adjusted) - not to exact scale

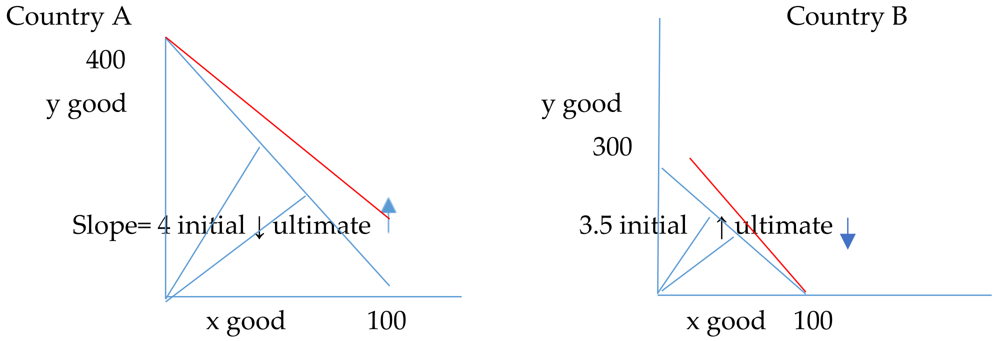
Utility Maximization



Utility maximization (above, and in Table 1 later) is assumed by calculating the areas beneath the points, the rectangles. This is done by 6] utility maximization = multiplying the x and y values for each point. A note should be made again that the smaller country is unable to satisfy the larger country. The red and black lines (the terms of trade with complete or partial specialization) make trading outside of the boxes of surety, signified by the dark purple lines, potentially more utility-maximizing than trading within the boxes. But, this gain is often not possible with a large country trading with a smaller one. If the smaller country cannot satisfy the larger one, then economic restructuring or expansion of the smaller country would have to occur. This assumes that the smaller country sees it as beneficial to come closer to its own initial ray of satisfaction. Or, the larger country could restructure its own economy. Another option would be for the countries to use partial specialization, as shown in figures 7 and 9. Utility for the smaller sized country is analyzed in part 6o, after analyzing the smaller countries below.

3.14. Math work for Utility Maximization with Similar-Sized Countries:

Figure 11 (#8 revisited) for Utility Maximization
The Ray of Satisfaction Used in Production



Here, the "exact" and "doubled" rays do not fall on the curves simultaneously. The "balanced" rays were provided earlier. In this example, "singular" rays do not fall on the red line curves for both countries, using 4, from the equation in Figures 7 and 9 "Average Rays": (53.33, 173.33) Country A, (46.7, 186.66) Country B. County A gains 53.33 x. So, it sells 186.66y, but B sells 53.3x for 186.5y. Yet, this number should add with the other country to equal the complete specialization of one country's y value. Thus, sometimes the balanced ray falls on curve, and sometimes it does not. Equally, sometimes the singular ray falls on the curve, and sometimes it does not. It depends on the sizes of the countries. A step-by-step process will be provided in Table 4, based on the author's observations of such examples.

6o. Math work for Total Utility of Differently-Sized Countries:

The following Table (1) lists utilities, starting with the highest sloped rays, for the larger countries in Figures 7, 9, and 10. Utility is the area underneath the graphs (again, with all of the assumptions previously discussed). And, using the red production lines, the Table then subtracts losses from being outside the "boxes of surety." These boxes are formed around the initial ray of satisfaction and the original blue production line. A more accurate assessment of utility is obtained, based on the consumers' original satisfaction. This loss to the "boxes of surety" is calculated as (100 – x) or (x – 100) times (x – 400) or (400- x) for Country A, and (50 – x) or (x – 50) times (y – 150) or (150 - y) for Country B.

For Figures 7, 9, and 10, observe the "total utility" in the table below (Table 1), or, at "total – loss," for being outside of the "boxes of surety," and "productivity loss" (explained later). Using 3.5 and 3.5 ("average" rays) is the highest value (54,466 for "total utility"). But, this is not possible given the size of the countries. A * *symbol* indicates if combinations of ray slopes are indeed possible. Using "exact" combinations of 3 and 4 are also not possible. Just as a note, the 300 extreme y value for Country B cannot be calculated, so a "counter-factual" estimate was used. A "doubled" ray with complete specializing is not possible, but using partial specialization yields a large amount of utility.

Using "balanced" rays (5.9 and 1.9) is the next highest combination, but it is also not possible given the countries' sizes. The next step (in Table 4), as we have seen, would be to look for a "singular" ray (8), which in this case works, or else, to take an "extreme" ray. For the larger country, the amount of goods traded shift more greatly with different rays. But, proportionately, the number of goods traded changes, and utility increases more, slightly, with small countries. With the last, second to last, and fourth to last row of Table 1, the use of partial specializing creates utility greater than at any possible specialized trading point. And, the points fall on the curves simultaneously. Without trade, a country would simply gain the consumption at the downward sloping blue lines (47,500), in Figures 7, 9, and 10. For this, the utility is shown by the third to last row in Table 1. A country does not need to start domestically where the blue and black production lines intersect: the start can come before, or after, the intersection, but production must be where they meet. In Table 1, as with example in section 3.7, the 3.5 ray creates the greatest utility, but the end points are outside of the "box of surety," so there is a loss.

Table 1.
Utility at Various Finish Points

Country A:		Country B:		A + B:		A + B:
Ray(finish points)=utility	Loss to Box:	Ray(finish points)=utility:	Loss to Box:	Total Utility:	Prod. Loss:	Total- Losses:
12.5 (50, 625) = 30,000	11,250	3.5 (50, 175) = 8,750	0	38,750	0	27,500*
8 (69.56, 556.6) = 38, 717	4,766.9	8 (30.4, 243.46) = 7,401.84	1,831.82	46,118.2	0	39,519.46*
4.87 (95.58, 465.47) = 44,489.8)	288.72	2.13 (62.2, 132.4) = 8,235.28	214.72	52,725.1	0	52,221.64
4.5 (100, 450) = 45,000	0	300 (1, 296.5): counterfactual = 296.5	7,178.5	45,296.5	0	38,118*
3.5 (114.24, 400) = 45,696	0	3.5 (50, 175) = 8,750	0	54,446	0	54,446
More Combinations	n/a	n/a	n/a	n/a	n/a	n/a
3 (123.07, 369) = 45,387	715.17	4 (46.6, 186.66) = 8711	197.64	54,098	0	53,185.19
4 (103.6, 412.5) = 42,735 (with partial specializing, starting at 775)	0	6 (37.25, 219.6) = 8,180.1	2,162.4	50,915.1	267	48,485.7*
4 (100, 400) = 40,000 (initial slope- no trade)	0	3 (50, 150) = 7,500 (no trade)	0	47,500	0	47,500

3.5 (110, 385), = 42,350, with partial specializing	150	3.5 (49, 171.5) =, 8,403, with partial specializing	21.5	50,753	240	50,341.5*
4 (102.88, 411.52) = 42,337.2, with partial specializing.	0	3 (53.85, 161.6), with partial specializing = 8,702.2	0	51,039.4	227	50,812.6*

Again, a * *symbol* here indicates if combinations of ray slopes are indeed possible, and balance trade.

3.15. Attempting Maximum Utility:

In Table 1, above, dealing with the larger countries in Figures 7, 9, and 10, and for all countries of similar size, then "exact" rays, followed by "doubled" rays, then by "average" rays will yield the most utility. But, all of these rays are unlikely to fall on both production curves. The exception is in using partial specialization. Even in this case, when they do not, then "balanced" rays will yield the most total utility, unless countries are of different sizes, and this ray is not possible given production. In this case, it is best to move to "singular" rays, or, if this is not possible, then to take the "extreme" rays that yield the highest calculated utility. These rays can be discerned and calculated from the green and yellow lines in Figure 9 for that particular example (this step-by-step approach is shown later in Table 4). One can see that a greater utility occurs where there is partial specialization, at slope of 4 and 6. A scholar might ask, though, why a country would not simply start production closer to their initial preferences.

Here, we must make one final assumption, that there is a utility "loss of production" caused by starting at an abbreviated, partial specialization starting point, due to loss of productivity of making less productive goods. This loss can be approximated by x and y of the starting points times the opportunity cost 60 x (800/200) and y start (0) x (300/100), summed = 240, in the 2nd 3.5 ray example coming. It is the difference between what a country can produce, and what it does produce. We also must relax the assumption that points do not fall below the Production Possibilities Frontier (PPF) (production curve) to simplify the math for maximizing utility. Even considering this, countries can still gain in utility with partial specializing. The equations below show how to calculate points using a 3.5 and 3.5 ray, which have given high utility values in every example, with partial specialization, which is the only type of trade possible with such "exact" rays. With partial specialization, the first step should always be to determine the finish points at which at least country wishes to end at after trade, using linear algebra. This is complex because countries' constants in their production equations depend on their starting points, and here it is that Country A's maximum of y goods is 775, or 97% maximization, or, 3% shifted, rounded. The proof of calculating the maximum utility, with partial specialization, will be left to future work.

Using finish points after trade to determine starting points with 3.5 rays to maximize utility:

Equations: [775 end/ (3.5 + 3.5)] = 110.7 * 3.5 = 387.5, and 339/ (3.5 + 3.5) = 48.3* 3.5 = 169.35.

Using a 3.5 ray with losses to productivity: we choose the ending points to be on the black curves, in Figures 7, 9, and 10. We use simultaneous equations, with partial specialization starting with the end point 775 for Country A: (110.7, 387.5) and (48.43, 169.53) - again, a 3.5 ray. Here, the * symbol means multiplying, in order to simplify. Again, starting points are based on how much countries decide to restructure towards their "end" points, which are the maximum amounts on the edges of the graphs, after a country decides to trade, but before it has done so. "Finish" points are after trade.

Unknowns: Axstart, Aystart, xtraded, Bxstart, Bystart; the underlines are for known quantities.

Proof of Theorem of Starting, End, and Finish Points

Axfinish – Ax start = Bxstart - Bxfinish

a) 110.7- Axstart = Bxstart – 48.43

Aystart- Ayfinish = Byfinish – Bystart

b) Aystart- 387.5 = 169.53 – Bystart

Axstart + Bxstart = 159.13

Aystart + Bystart = 557.03

775- Aystart = 3.5Axstart

339.06 – Bystart = 3.5Bxstart

Axstart + [(339.06 – Bystart)/3.5] = 159.13

-3.5Axstart = -775 + 549.8 + Bystart

3.5Axstart + 339.06 = 3.5 (159.13) + Bystart

0 + 339.06 = -775 + 549.8 + 556.955 + 2Bystart

Bystart = 3.653

Aystart = 557.03 – 3.653 = 553.38

Bxstart = (339.06 – 3.653)/3.5 = 95.83

Axstart = 159.13 – 95.83 = 63.3

Starting Points

Country A	Country B	Country A	Country B
(63.3, 553.38),	(95.83, 3.653)	(110.7, 387.5) and	(48.43, 169.53):

and trade is somewhat balanced, and the ending points are reached.

▫

6.15.1. Penultimate Trade Equation:

In the equations below, the term "end" means the maximum point of production after the country specializes. Examples are 800 + 200, or 300 and 100, for Countries A and B with Figures 7, 9, and 10. In these cases, Byend is 350 - 3.125% * 350**.** This is because the production equations change depending on their starting points (before trade). So long as production is at/ below/or before the intersection of the black and blue production lines, according to the desired amount of "shift" in production, discussed in Section 6b, then production using partial specialization is possible.

Penultimate Trade Equation:
7] Bystart = [-Ayend + Ayfinish + Byfinish + t.o.t. (Axfinish + Bxfinish) – Byend]/-2

By plugging in to the simultaneous equations above, all of the variables can be solved.

Having used partial specialization, each country comes closer to maximizing its utility.

3.16. Observations on the Penultimate Equation to With Intersections:

In the last example, the start and end numbers are slightly skewed- the x and y finish numbers do not add up to the end points, and Equation 7 does not balance. The math, with the "shifts" and starting points, also needs more elegance, shown later in section 3.17.1. In the last example, an end point of 775 was used for Country A. This equation is correct, but it is apparent that the end points and the "shift" must take into consideration the "intersection" of the partially specialized production line with the original blue production curves. As explained above, this is in order to be sure that that the country's starting points can intersect the blue production curves, which they do not above. This way, the countries can produce the desired combination of goods necessary for the chosen end points.

Otherwise, as in the examples above, the country would have to produce "extra" of the specialized goods to reach its end points. The starting points would fall on the black production line at an x point for Country A greater than the intersection with blue curve, needing extra production. This production, in the preceding examples, might be possible, but one cannot be sure of the resources needed to create such production. In order for the black production curve of partial specialization to intersect with the blue production curve, in our example for the large country, the initial "shift" can only, at its most, lower the Ay end point to 750, or 6.25% of specialization, discerned form Figures 7 and 9, in Country A. This shift would ensure that the end points are at least equal to, or above, the blue end points within the "box of surety." Still, the country can gain, if the gains are greater than the assumed losses. The most that Country B can shift is 50, and the simplest endpoint is x = 100, which falls on Country B's blue curve, as well. In these cases, the Ayend, Ayfinish, and Axstart points can be calculated, after choosing the amount of "shift." The Ax intersection equation is given by:

8] Axintersection = (800 – shift)/ (original slope/t.o.t.), and:

Axfinish = Axintersection + 3.5x *
Ax finish = shift/ (original slope + t.o.t.) where the shift is equal to or less than the max end point.

*In this case, albeit, it is possible for Country A to "shift" more than 750. This occurs since the black production curve falls below the "box of surety," and the intersection point as well, in order to reach a desired end point, such as in 3.5 ray example. The country can still gain via trade, and increase its number of goods, and possibly overall utility, but the end points will well exceed the "box of surety." The further the space between the terms of trade and the original country's slope, the more specializing that is possible. For the maximum shift of Country A, the points with the "box of surety" can be as little as simply meeting the blue production line, or it can be as much as 6.25 points higher than the blue line equilibrium of 100 in the x value: maximum shift= [(800-750/800]. For Country B, it can also shift more than 100-50 = 50: the % change is greater, since it is a smaller country. The change is given by:

9] Shift= {MaximumX – [(midpointX + midpoint)/t.o.t.]}/100
100 – [(50 + 150/3.5]/100 = 7.2 %

In order to calculate the Country A's "intersection," "end," and "starting points," further equations are needed. This is because it is a case where the intersection occurs outside of the "box of surety," and the starting and end points do as well. The additional equations are:

10] (Axfinish – Axintersection) = Axintersection * (t.o.t. + original slope)
11] Desired xfinish * (t.o.t. * ray) = Axend
110.7 * 7 = 773.5, and it does indeed intersect the blue curve in the "box of surety,"
since (800 – 773.5)7.5 = 54, the Aystart and intersection point.

3.17. The Penultimate Equation with Intersections Examples:

In this five part section, we now refine the prior equations, and use steps, to find starting points with the "intersections," and the desired "shifts." We compare the results with: the previous section, with rays of 3.5 and 3.5, and with "exact" rays, whereas "intersections" were not used previously.

With the 3.5 rays' example, it is helpful to start with the smaller country. So, we use Country B, and a chosen shift to 339.06, the finish point. With simultaneous equations= 339/7= (48.4, 169.5). So, Country A then shifts by a "similar" amount, and the percentages of the two shifts are approximately 1.14% different (4/3.5) in percent. They are also Δ800/Δ300 different in actual numbers. This statistical discrepancy will be left for future scholars. It is hypothesized that the shift allows the t.o.t. to stay in-tact, discussed with Figure 3. Also, as stated, this paper uses rounding of decimals to different places.

3.17.1. Combining equations, with steps:

12] <u>800 – (shift)</u> = 1/[(CountryAstartslope – t.o.t.)] x shift + CountryBxstart – CountryBxfinish

(3.5 + 3.5), the ray of satisfaction plus the t.o.t.

where By shift = Bxshift x 3.5 = (350 – Byshift)/3.5 +3.5
and where: 1/(4-3.5) = 2

$$\frac{(800 - \quad shift)}{(3.5 + 3.5)} = \frac{2 \text{ x shift}}{1} + (100 - \text{Bxshift})$$

Table 2; A series of steps are given, from start to end, to determine the rays and trade amounts:
Proof of Theorem for Equations 7-12

1: Determine the Country B end and start points, for x and y, and the amount of shift, in either order. Or, Country A could be started with, but that approach is not used here: No shift, y and x are 350 and 0. Country Bx starts at 350/(3.5 + 3.5) = 50, x 3.5 = 175
2. Make sure that these points fall of the blue production lines. Yes, because the blue line intersects the red line along the x axis.
3. Calculate the x start points for Country A and B, using Equation 12. 800 – 7 (shift) = 15 shift + 7 (50) Shift = 30, x 2 = 60
4. Calculate the Country A shift, end points, and starting points. x start = (800 – 30)/(3.5 + 3.5) = 60 x finish = 50 +60 = 110, x 3.5 = 350y Starting Points Finish Points Country A Country B Country A Country B (60, 560) (100, 0) (110, 385) (50, 175)
5. Check via addition and/or subtraction. 560- 385 = 175
6. Check using equation 7. Bystart = [-Ayend + Ayfinish + Byfinish + t.o.t (Axfinish + Bxfinish) – Byend]/-2 0 = [-770 + 385 + 175 + 3.5 (110 + 50) – 350]/ -2 = 0: yes! Trade balances.

▫

3.17.2. With 3.5 rays and Country B Partially Specializing:

By assuming a different start point, and a different amount of shift, it is still possible for both countries, including Country B, to "shift." They reach a suitable "intersection," whereby Country B would still fall on the blue line. This can be done without having to start at its maximum x end point, and by using: Country Bystart = 300 – 3 (Country Bxstart). Then, using Equation 12, more fully detailed:

Proof of Theorem for Equation 12

(800 - shift) = 1/(4-3.5) x shift + Bystart – Byfinish (3.5 + 3.5) 1
(800- shift)/7 = 2 x shift + CountryBxstart - CountryBxfinish, with: Country Bystart = 300 – 3 (Country Bxstart), or 300 – 3 (100 – 2)) = 6.
In the example from 6p: 800 – 7 (51.6) = 15 shift, But, if Bx shifts to 2 for Bystart, above, for example: The change is that CountryByend shifts to 343, using 350- [(100 – 98) x 3.5], and: Bxfinish becomes = 343/7 = 49
For Country A, y = 800 - 7 (98-49) = 15 shift. Shift= 30.47 x 2 = 60.93 Starting Points: Decided first Country A Country B Finish Points (60.93, 556.27) (98, 6) (110, 385) (49, 171.5):

▫

3.17.3. Observations:

All of the finish points fall on both black and blue lines, and balance using Equation 7. In both examples from section 3.17, the Country A starting points are, correctly, slightly less than in part 3.15; the "intersection" results in *less* loss of productivity. The finish points for Country A are slightly different than Figures 7 and 9 because the start points are different. These numbers might also have statistical discrepancies because the "shifts" do not occur in equal percentages between the two countries. The maximum "shifts" are, in Country A, 6.25%, and the maximum "shift" in Country B is 7.2%, as already explained. The relationship (by division), again, is 1.14, which is the slopes of 4/3.5= 1.14. The derivation for how these shift differences affect the statistical discrepancy is, again, left to others. Furthermore, in order for countries of different sizes to trade, and reach "exact" rays, it is likely that an "intersection" can be found. Still, this intersection may not always be possible. Therefore, the starting points may have to differ in terms of the "shifts," or percentages of shifts. Or, there need to be some adjustment of the countries' resources in order to produce more of the specialized good for a starting point not on the blue lines, in combination with the goods traded for.

3.17.4. Finding "Exact" Rays with Intersections and Steps:

With the "exact" rays example: - the end points are calculated using Figures 7 and 9, with a 0 shift that has already been chosen for Country B, so a 350 maximum y end point. The less the partial specialization, the higher the points will be when the black line passes through the "box of surety." The previous, 3.5 ray example works well with equation 12, because with the same ray for each country, each country gains the same proportionately. This may not always be so, as below, with "exact" rays, Country A's finish points are slightly less than in Figures 7 and 9. So, finding points with "exact" rays:

Table 3: Finding Exact Ray Start and End Point Steps:

Proof of Theorem of Steps

1: Determine the Country B end and start points, for x and y, and the amount of shift, in either order. No shift, y and x are 350 and 0. Country Bx starts at 350/(3.5 + 3) = 53.85, x 3 = 161.54
2. Make sure that these points fall of the blue production lines. Yes, because the blue line intersects the red line along the x axis.
3. Calculate the x start points for Country A and B, using Equation 12. (800 – shift)/(3.5+4) – 2 shift – 46.15 = 0 (800- shift)/7.5 = (2 shift + 46.15) shift = [800 – 7.5 (46.15)]/16 shift= 28.37, x 2 x start = 56.73
4. Calculate the Country A shift, end points, and starting points. 56.73 + 46.15 = 102.88 or, (800 – 28.37)/7.5 = 102.88, slightly less than 103.6 in Fig. 7, because of a greater shift. y = 800- (4 x 56.73) = 573.08 Starting Points — Finish Points Country A Country B — Country A Country B (56.73, 573.06) (100, 0) — (102.88, 411.52) (53.85, 161.54)
5. Check via addition and/or subtraction. 573.06 – 411.52 = 161.54 = 0 = yes!
6. Check using equation 7. Bystart = [-Ayend + Ayfinish + Byfinish + t.o.t. (Axfinish + Bxfinish) – Byend]/-2 0 = [-771.63 + 411.52 + 161.54 + 3.5 (102.88 + 53.85) – 350] -2 = 0 = yes! These are the answers to our paper-wide search!

▫

3.18. Overall, Entire Paper Step-by-Step Approach:

The following Table 4 shows the *overall* steps that should be taken, based on the examples in this paper, for obtaining rays of satisfaction which maximize overall utility based on consumption consistency. A complete mathematical proof of utility is left to others.

Table 4.
Steps to Maximize Utility

Step 1	Decide on partial or complete specialization.	Once completed, then…
Step 2	Take "exact" rays.	If not possible, then…
Step 3	Take a "doubled" ray	If not possible, then…
Step 4	Take an "average" ray.	If not possible, then…
Step 5	Take a "balanced" ray.	If not possible, then…
Step 6	Take a "singular" ray.	If not possible, then…
Step 7	Take an "extreme" ray	If not possible, then…
Step 8	If using Complete Specialization, then now use Partial Specialization in some form (using the 6 steps from 6r, especially for "exact" rays for utility maximization), and repeat the process, or adjust the terms of trade. Otherwise, there is no beneficial trade possible between the two countries, except for relaxing the "box of surety" or consumption-consistency assumptions.	This is the last option.

3.19. (final results). Additional Example Using All Sections:

This section provides an additional example using "medium" sized countries, and how to maximize utility, based on the previous examples, and the step-by-step approach in Table 4.

Figure 12
Medium-Sized Countries

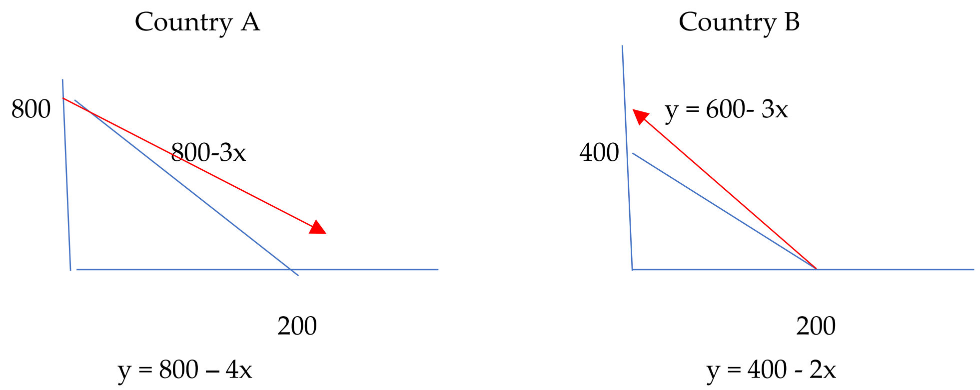


Table 5: Final Example- Using Steps from Table 4 to Maximize Utility:

1.	It is randomly decided to start with specialized trade, using a terms of trade of 3.
2.	(114.3, 457), (120, 240). Trade does not balance.
3.	(114.3, 457), (85.7, 342.8). Yes, a "doubled" ray balances trade on the red lines.
4.	Even if continuing, (133.33, 400), (100, 300) does not balance.
5.	Even if continuing, [6/(800/600) + 1)] = 2.57, [(6/(600/800+1)] = 3.42, does not balance.
6.	If continuing, 800/200 = 4, so, (114.3, 457), (85.7, 342.8) works as a "singular" ray.
7.	Not needed.
8.	Not needed.

4. Discussion:

Empirically, in 2021, the U.S. traded $4.5 trillion dollars' worth of goods, with $1.7 trillion dollars of exports, and $2.8 trillion dollars of imports, yielding a trade deficit of $1.1 trillion dollars, the most it has ever been. This deficit is truly most dangerous long-term, due to pressures on the currency, and an imbalance of saving, but not immediately as it simply represents the present preference of consumption versus and saving and investing, and rather reflects the U.S. budget deficit. During the fear of the Covid-19 pandemic in 2020, and supply-chain issues, trade was lower at $3.75 trillion dollars, with $1.4 trillion dollars of exports, $2.3 trillion dollars of imports, and a trade deficit of $902 billion dollars. Before the pandemic, trade had been fairly constant at approximately $4 trillion per year, with an approximate deficit of $700-750 billion dollars ("Trade in Goods" 2022, 1-4). Most of this American trade is with Mexico and Canada due to their geographic proximity, as well as with China, Japan, Germany, and the rest of Europe, due to their sizes and different types of goods produced, often at greater productivity for these goods with lower labor costs, resulting in less expense to the U.S. consumer.

Figures 7 and 9 are perfect examples of how partial specialization allows both countries to come closer to their individual rays of satisfaction of their countries' consumers. Because the small country cannot fully satisfy the large country, it would need to restructure by producing more of its specialization good. Instead, an alternative is to use partial specialization. For Country A, partial specialization may result in a new starting point on the production possibilities curve such as at (50, 625) ($y = 800 - 3.5x$), in Figure 7, in which case the production curve, and ending consumption, still shift markedly outwards. Albeit, the new ratio of goods consumed, compared to complete specializing, is less, and may fall outside of the "box of surety." However, the country need not produce all of the extra specialized goods to trade, and reduce the non-specialized goods. Utility, in sum, is shown in Table 1. In some cases it may be better for countries to not even to trade, unless it is done with partial specializing.

Historically, some economists, historians, or politicians might argue that the United States may have exported capital into China and other South East Asian countries, starting in the 1970's with United States President Richard Nixon's trip to China, in order for these nations to supply the United States with less expensive goods. The United States, meanwhile, focused on manufacturing and technology. Many scholars and leaders in different countries have also made the case for numerous years since that era for economic restructuring in favor of certain industries, such as technology, within their countries, in order to create "better" jobs. For another example of this, in the 1960's, Japan started reorienting its economy towards its trade sector, heavily based on electronics, autos, and technological goods, which inevitably must have come with empirical economic costs. But, if an imagined island is made mostly of coconuts, and trade is for a few pineapples, yes, the pineapples will be highly demanded, but most likely, the food culture will still rely on coconuts, the staple, or "Giffen" good, such as rice with Japan.

In the United States, there is also the constant push for "good paying jobs," which are usually more high-tech in nature, and thus presumably higher paying. According to Amadeo (2021), the United States currently specializes in chemical, banking, aerospace, computer parts, and defense parts, and the U.S. has transitioned away from consumer products to these more high-tech specialized goods over the past several decades (Amadeo 2021, 3). One might add that the United States also has a very successful pharmaceutical industry. Amadeo also writes that, for another example, Middle East countries are not just good at producing oil, but also chemicals derived from oil, which help fuel manufacturing in Southern Europe (Amadeo 2021, 1). Services can be traded, as well, by sending accounting and financial paperwork overseas, for example. Education, though, must be readily available for such restructuring.

But, does this trend towards specialized products necessarily mean that lesser-paying jobs, for less skilled individuals, cannot exist, too, alongside such jobs which require higher education, which not everyone may be able to obtain and afford? Cannot there still be a middle class? Why do some small industries, such as furniture stores, the arts, or basic manufacturing, have to die, and not co-exist? As shown here, many small businesses and many small sectors can remain in countries, and flourish, even though the country moves towards specialization. Futuristically, many say that Artificial Intelligence (AI) (super computers) will eliminate jobs, which is largely true, but will not such technology need upkeep for new jobs, and entrepreneurship for new uses, thereby yielding new fields for partial specialization?

In the 2000-2001, United States' short recession, the cause was various factors, among them the after-effects of the Dot.com bubble bursting, lack of confidence in political and economic leaders, uncertainty over future expectations, and corporate debt scandals involving Enron and World.com. In addition, scholars have found that economic restructuring was also a possible cause. Figura and Wascher (2008) estimate the loss from the restructuring at this time to have been 0.5% to 1 % per year of Gross National Income (GNI), and doubtlessly also to Gross Domestic Product (GDP). With both, not only the nation loses, but also individual income earners. Lower productivity often comes from restructuring, they find, because firms and countries change from the goods that they are producing.

With the Covid-19 pandemic, economic restructuring, which caused more goods to be bought online, was not a choice. The pandemic resulted in structural employment, and largely changed the ways employees are working. But, restructuring often is a choice. With restructuring, the loss is more to small firms, as large international companies, instead, can diversify and produce many types of goods in many divisions of their companies. Understanding foreign regulations, developing new supply chains, switching types of physical capital, maintaining company image, and keeping worker morale high are all needed to restructure an economy towards its trade sector, which might involve laying off workers, closing plants, moving operations, and taking other actions that cost an economy. Covid-19 and the Green movement's response to climate change, have and will continue to create new issues in restructuring, possibly leading to short-term structural unemployment. But, in the long-run possibly increasing utility if the new firms and jobs are part of industries that are partially specialized

In order to restructure, the cost of restructuring (C) must be less than the opportunity cost of more goods obtained in trade (OC), which is also the benefit of restructuring.

Equation [10]: Restructuring Cost < Opportunity Cost (OC) (or Benefit from Specialized Trade).

Table 6.

Costs to Restructure (not all-inclusive):

Cost of new labor/retraining
Cost to obtain and use new natural resources/parts
Cost to obtain new capital (property, plant, equipment)
Cost of relocating businesses
Cost (interest) to obtain new financing
Cost of developing new technology
Cost to develop new supply chains ("finding" new trade partners)
Cost for new marketing, and finding new consumers
Costs to small countries in scaling production
Cost from competition causing lesser firm profits
Cost to consumption consistency (excess inventory, changing production)
Cost to utility from exceeding the "box of surety" (forcing consumer's habits)
Cost in risk (cyclicality, over-concentration, or economic "shocks")

- Created by the Author. Many of these costs are assumed away by Ricardo's model.

This model becomes even more useful if we change Ricardo's assumption from a two good model, to a model in which the non-specialized good represents an index of all non-specialized goods. In this sense, we can make more realistic judgments about countries' entire economies in terms of restructuring. While neo-classical economics assumed diminishing returns, the graphs here assume that there are losses to countries in production of specialized goods, but even without these losses in Table 1, the math shows that countries still can maximize their utility through partial specialization, rather than restructure. According to Freedman (2018), a global analyst, the 2006-2007 time frame, before the United States 2008 financial crisis, was a time where a lot of restructuring was taking place, but Freedman does not indicate if this had a direct effect on the crisis, or if it was a symptom of the oncoming recession. Many firms were trying to improve their balance sheets by storing more in cash (Freedman 2018, 1-2).

Freedman (2018) says that this restructuring was also occurring at the time of his writing, 2018, in the United States and in countries around the world. Much of the restructuring was, and is, a result of years of corporate mismanagement, he writes. Restructuring is usually cyclical, he notes, occurring around the times of economic problems, citing Irwin Gold, a former top finance executive. Specific firms that are now restructuring include Malaysian Airline, Toys R Us, and many telecom, appliance, and car making firms. Global trade was strong before the pandemic, but corporate debt was at the highest levels since the 2008 financial crisis, so many companies are now restructuring their financing mixes, having implications for their countries, particularly in the United States, Australia, and China, he writes. Some of the countries around the world that have firms that are restructuring may need massive loans from the International Monetary Fund (IMF) or the World Bank to do so (Freedman 2018, 1-2).

An excellent overall example of partial specialization to conclude this paper is to take the example from Ricardo himself. Consider the United Kingdom (U.K.) vs. Portugal- while the first country only made cloth, and the latter wine, the two now currently each make both. The U.K. makes just over 5 billion euros worth of cloth, while Portugal makes about 2.5 billion, as of 2019 (Liu 2022, 1). For wine, Portugal makes nearly 20 million bottles per year, and the U.K. makes some 15.6 million bottles ("Wine From" 2022, 1). Thus, it is possible to have different sectors within a country, while still playing to a country's strengths. Just to add to the discussion, similarly, a model was presented by the famous economist Arthur Lewis in the 1950's (1954) for equal production of agriculture and manufacturing in developing countries, akin to the concepts presented here, although Lewis' work focused on the interchangeable skills of labor, and how this impacted production of different goods, whereas the work here did not focus on resources used in production but on consumer preferences, to maximize utility.

5. Conclusion:

In summation, assuming "consumption consistency" should allow researchers and policy-makers to better find amounts of trade that maximize a country's utility, both from its starting ray of production, through its ending ray of production after international trade.

Meanwhile, using "partial specialization" has proven to be a useful theory that can allow countries to come closer to their desired ending points (maximum production amounts) and finish points (amounts after trade) without having to entirely restructure their economies. As explained beforehand, each country has a trade "preference" which orients towards their ray of satisfaction. The consumption consistency assumption is that the closer to the points on the production possibilities curve (the downward blue, black, and red lines in the figures) is to the initial consumption preferences, given by the rays of satisfaction, and extended through trade, is where utility is theoretically maximized.

The primary example in this study used a large and small country trading. With this size imbalance, and using one country's ray of satisfaction rather than the others', one country ends up supplying massive amounts of one good to another. The solution for this dilemma, as already suggested by Barry (2018), and as we have seen here in depth, is to partially specialize, and/or to take various ray of that yields certain production, and then consumption, points. This occurs only if the points are possible mathematically in the amount of goods traded, given the countries sizes, such that a ray produces points that fall on both countries' production/consumption lines. Some of the rays suggested were an "exact" ray, a "doubled" ray, an "average" ray, a "balanced" ray, and a singular sloped, "singular" ray. If this still does not occur, then trade between the two countries will not happen, or countries can try to steer trade to favor its ray and "preferences," through an "extreme" ray. Or, the terms of trade may need to be changed, or partial specialization turned to. A final equation for proving total utility is left to others.

Countries, though, do in fact try to obtain terms of trade closer to giving up fewer of their preferred goods, noted here with brief empirical evidence, and, according to many texts, such as in Salvatore (1996), Sawyer and Sprinkle (2009), and Kreinin (2010). In such texts, as already alluded to, larger nations often use their market power and supply and demand to steer the terms of trade in their direction. Whether or not the average rays of satisfaction must be exactly equal to the terms of trade to maximize utility, as was in this case, will be left to future research. But, a greater terms of trade should lead to steeper rays of satisfaction, and benefit the smaller country more. Table 1 shows the most utility, that can be realized, at partial specialization points for the example in Figures 7, 9, and 10, but with a the formula for finding points for total maximization with partial specialization, which would occur at each country's individual production ray, an "exact" ray, though the exact mathematical proof applying Calculus and/or spatial geometry to the graphs will be left to others. Consumption consistency also reflects how countries prefer to spend their income. In fact, Schotter (2009) wrote about the ray of satisfaction, what he termed the "Income Expansion Path," as increasing, proportionately between two goods, via increases in income. Whether or not this ratio continues, empirically, with increased national income, is also an interesting question for future researchers.

With these theories in mind overall, though, it need not be that each country has to focus on producing one sole good, or trading for the other's countries' specialized good, or that it is necessary to focus solely on one occupation in order to gain. While such sole a focus might result in the "most" apparent gain, there may be costs for an economy of switching to very few production sectors, which was addressed in the discussion section. Instead, consumption consistency and partial specialization of countries can have positive benefits. Even without trading as many goods, partial specialization can still yield a high indifference curve, and maintain a close ratio of goods ("consumption-consistency") to that which consumers prefer domestically. Countries can then make small adjustments, such as with the blue crescent, shown in Figure 7. Partial specialization is especially helpful with countries of different sizes; and, future research should also look into this issue when multiple trading countries are involved.

Further research should also look for more empirical examples of partial specialization occurring, such as whether or not it occurs within different industries, and whether states overall try to maintain a balanced ratio of goods. This would be in accordance with the theory presented here. The amount of goods that countries "should" trade, using "consumption consistency" to meet consumer tastes- and use partial specialization to obviate unnecessary restructuring- should be equal to the amount of goods that countries "do" trade. Other factors, though, why nations trade with other ones are based on: political orientations, geography, culture/ language, risk, and many other variables that could be tested in the future using statistical regression. This could include a variable representing comparative advantage representing the preferences, in ratios, of domestic consumers.

Perhaps the most under researched of these variables is risk, for which a viable equation could be created that links partial specialization to diversification and to the equations of Barry (2020), as already noted. Useful for this possible equation is the fact that countries "shift" to specialize to a certain percentage. Future research should also analyze the losses if and/or when initial production (the "shift") falls below the Production Possibilities Frontier, or if countries' starting points of trade with partial specialization are beyond their points of intersection with initial production curves, and how this affects the future gains from trade. Also, the models here assume that final points after trade using partial specialization could not be obtaining by consuming different mixes of goods on the indifference curves with complete specialization, albeit with very far-stretching indifference curves. This might thereby lessen the need to partially specialize. Still, the fact that the penultimate equation works by percentages (and a proof of this is needed) should make decisions for policymakers easier. Countries should look to trade with other countries that have, or will form, trade industries in proportion to the percentages of trade, or "shifts," that are equal to, or near, their own.

Finally, future work should also simplify the math here to a more refined and elegant condition, and it should use more examples. It should look for small-or-large country anomalies whereby the rules here do not hold. Future work should also use more complex, neo-classical models, so that determining the exact quantity of goods a country should trade with diminishing returns, due to depreciation or the reallocation of resources, can be identified in order to yield even more accurate indifference curves. Using Calculus as such with neo-classical curves, and not just matrix and linear algebra, would also be helpful to doing this, for applied realism. As with all models, many assumptions are needed, and this paper has in-fact attempted to use these assumptions, and to reassess other ones that might be specious, in finding how many goods conceptually are produce and traded to maximize utility, which increases happiness for the world. By continuing to refine these Ricardian assumptions, in total, would continue to aid in government trade policies and business decisions for all countries concerned.

In short, the ideas presented here are believed to be entirely original, with no similar outstanding work discernable. If there is research on this topic, however, the graphs and equations here have certainly been a unique way of portraying the information in ways that are easier to conceptualize. In this way, it contributes noticeably to the existing theoretical literature on international trade, by showing that trade need not be specialized completely, and that countries maximize utility by producing and trading close to their domestic preferences, if possible, whatever those preferences may be, which were assumed here with new ideas. To sum, future work could add in mathematical eloquence and provide more real world examples and evidence, which are left to others. However, the works of Hume (1700s, 1955 ed.), Ricardo (1817), Edgeworth (1877), and others have been used and added to by applying a different methodology of graphs, and equations, using matrix/linear algebra, building on the lines of Ricardo's theory of comparative advantage.

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